MIGRATION-VELOCITY ANALYSIS FOR TI AND ORTHORHOMBIC BACKGROUND MEDIA

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CASP Project[1]

ANALYSE DE VITESSE PAR MIGRATION DANS LES CAS DE MILIEUX ENVIRONNANTS TI ET ORTHORHOMBIQUES

Il est essentiel de connaître le modèle de vitesse du milieu environnant pour effectuer la description précise du réservoir que l'on veut obtenir par imagerie et par inversion. Les méthodes conventionnelles permettant de reconstituer le modèle de vitesse de fond, telles que les méthodes de vitesse migratoire, supposent souvent une sous-surface isotrope et peuvent donner des descriptions de réservoirs manquant de précisions lorsque la sous-surface contient des formations de roches anisotropes. Nous généralisons ici le concept de vitesse migratoire en permettant à la vitesse de fond d'être transversalement isotrope (TI) par rapport à l'axe vertical ou orthorhombique.

Cette méthode consiste à scanner différents modèles de vitesse anisotropes à l'aide d'une migration à décalage de phases et de sélectionner des paramètres anisotropes sur la base des variations d'amplitude des résultats migrés (analyse par focalisation).

Sachant que le modèle de vitesse de fond anisotrope est généralement décrit par plusieurs coefficients d'élasticité, il importe d'adopter une procédure de scannage efficace. Nous avons choisi de travailler sur des sections azimutales communes.

Pour une section azimutale commune donnée, nous avons effectué une analyse continue de deux paramètres : la vitesse normale de déplacement et le paramètre anisotrope connu sous le nom d'anellipticité. Ces deux balayages nous permettent de reconstituer un modèle de vitesse azimutalement isotrope. L'opération est ensuite répétée pour différentes sections azimutales communes ; chacune d'entre elles donne un nouveau modèle de vitesse azimutalement isotrope si le support est azimutalement anisotrope. Le nombre de sections azimutales communes, et par conséquent le nombre de modèles de vitesse azimutalement isotropes nécessaires pour reconstituer un modèle de vitesse azimutalement anisotrope, dépend du type de symétries. Par exemple, il faut seulement trois sections azimutales communes pour un support orthorhombique.

Étant donné que la contribution d'élément isotrope dans la majorité des formations de roches est généralement plus importante que l'élément anisotrope, nous avons proposé de baser le relevé des valeurs d'anellipticité en défalquant le résultat de la migration isotrope de celui de la migration TI au lieu d'utiliser directement les résultats migrés TI.

MIGRATION-VELOCITY ANALYSIS FOR TI AND ORTHORHOMBIC BACKGROUND MEDIA

A knowledge of the background velocity model is crucial to achieve the accurate reservoir description now expected from imaging and

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inversion. Conventional methods for reconstructing the background velocity model, like migration-velocity methods, often assume an isotropic subsurface and can yield inaccurate reservoir descriptions when the subsurface contains anisotropic rock formations. Here, we generalize the migration-velocity concept by permitting the background velocity to be transversely isotropic (TI) with respect to the vertical axis or othorhombic.

The scheme consists of scanning different anisotropic velocity models using a phase-shift migration and of picking anisotropic parameters based on amplitude variations of migrated results (focusing analysis). As the anisotropic background velocity model is generally described by several elastic coefficients, it is important to adopt an efficient scanning procedure. We have chosen to work with common azimuthal sections.

For a given common azimuthal section, we sequentially scan two parameters: normal move-out velocity and the anisotropic parameter known as anellipticity. These two scans allow us to reconstruct an azimuthally isotropic velocity model. The procedure is then repeated for different common azimuthal sections; each common azimuthal section leads to a new azimuthally isotropic velocity model if the medium is azimuthally anisotropic. The number of common azimuthal sections, and therefore the number of azimuthally isotropic velocity models, needed to reconstruct an azimuthally anisotropic velocity model is dependent on the type of symmetries. For example, only three common azimuthal sections are needed for an orthorhombic medium.

As the contribution of isotropic component of most rock formations is generally more important than the anisotropic one, we have proposed to base the picking of values of anellipticity on the subtraction of the result of isotropic migration from that of TI migration instead of using directly the TI migrated results.

ANÁLISIS DE LA VELOCIDAD MIGRATORIA DE FONDO TI Y ORTORÓMBICA

Es de primordial importancia conocer el modelo de velocidad de fondo para proceder a la descripción precisa de un yacimiento, cuando se desea obtenerla por imaginería y por inversión. Los métodos convencionales que permiten reconstituir el modelo de velocidad de fondo, como por ejemplo los métodos de velocidad migratoria, presuponen con frecuencia una subsuperficie isótropa que puede proporcionar descripciones de yacimientos que carecen de precisiones cuando la subsuperficie contiene formaciones de rocas anisótropas. Se generaliza en este artículo el concepto de velocidad migratoria al permitir a la velocidad de fondo ser transversalmente isótropa (TI) con respecto al eje vertical u ortorómbico.

Este método consiste en analizar (por escáner) diversos modelos de velocidades anisótropas por medio de una migración con desplazamiento de fases y seleccionar los parámetros anisótropos en base de las variaciones de amplitud de los resultados emigrados (análisis por focalización).

Al saber que el modelo de velocidad de fondo anisótropo se describe por lo general mediante diversos coeficientes de elasticidad, es importante adoptar un procedimiento de análisis eficaz. Por este motivo, el autor ha optado por trabajar según secciones azimutales comunes.

Para una sección azimutal común determinada, se ha efectuado un análisis continuo de dos parámetros, o sea : velocidad normal de desplazamiento y parámetro anisótropo por la denominación de anelipticidad. Estas dos exploraciones permiten reconstituir un modelo de velocidad azimultamente isótropa. Acto seguido, se repite la operación para diversas secciones azimutales comunes. Cada una de ellas permite obtener un nuevo modelo de velocidad azimutalmente isótropa si el soporte es azimutalmente anisótropo. El número de secciones azimutales comunes, y por consiguiente el número de modelos de velocidad azimutalmente isótropos, necesarios para reconstituir un modelo de velocidad azimutalmente anisótropo, depende del tipo de simetrías. Por ejemplo, únicamente se precisan tres secciones azimutales comunes para un soporte ortorómbico.

Habida cuenta que la contribución de un elemento isótropo en la mayor parte de las formaciones de rocas es generalmente más importante que el elemento anisótropo, el autor propone tomar como fundamento el registro de los valores de anelipticidad deduciendo el resultado de la migración isótropa de aquel de la migración TI en lugar de utilizar directamente los resultados emigrados TI.

INTRODUCTION

The reconstruction of anisotropic background velocity models is probably the most important challenge in seismic data processing with anisotropy. The challenge is two fold:

- to distinguish between variations of physical parameters which contribute to the observed reflected seismic field as inhomogeneities and those which contribute as anisotropy;
- to develop methods for reconstructing separately these two components of the background medium. For a general solution to this problem, the background medium must be considered arbitrarily inhomogeneous and anisotropic in its elastic behavior.

The difficulties in considering such a background velocity model are compounded by the multitude of independent degrees of freedom it may possess. It may have as many as 21 independent elastic coefficients, and each of these coefficients may vary as a function of position in the subsurface. It becomes clear that the demands on any general background reconstruction are immense. However, methodological advances can be made using more simple models of the background, like transversal isotropic (TI) with respect to the vertical axis or orthorhombic models. My objective here is to derive a method for reconstructing these two particular cases of anisotropic background models. This method generalizes the migration-velocity method by replacing the isotropic background with an anisotropic one. I will use prestack phase-shift migration.

The migration-velocity method consists of scanning over different velocity models using prestack migration, then using focusing analysis to construct an improved velocity model (Fig. 1). To apply this concept to anisotropic velocity models, the migration algorithm must be reformulated to include anisotropy and a new scanning procedure must be introduced to accommodate the occurrence of several coefficients describing the anisotropic velocity models. These are the two main tasks of this paper.

In spite of the assumption that the background velocity model is only depth dependent (or homogeneous), the prestack phase-shift migration remains a

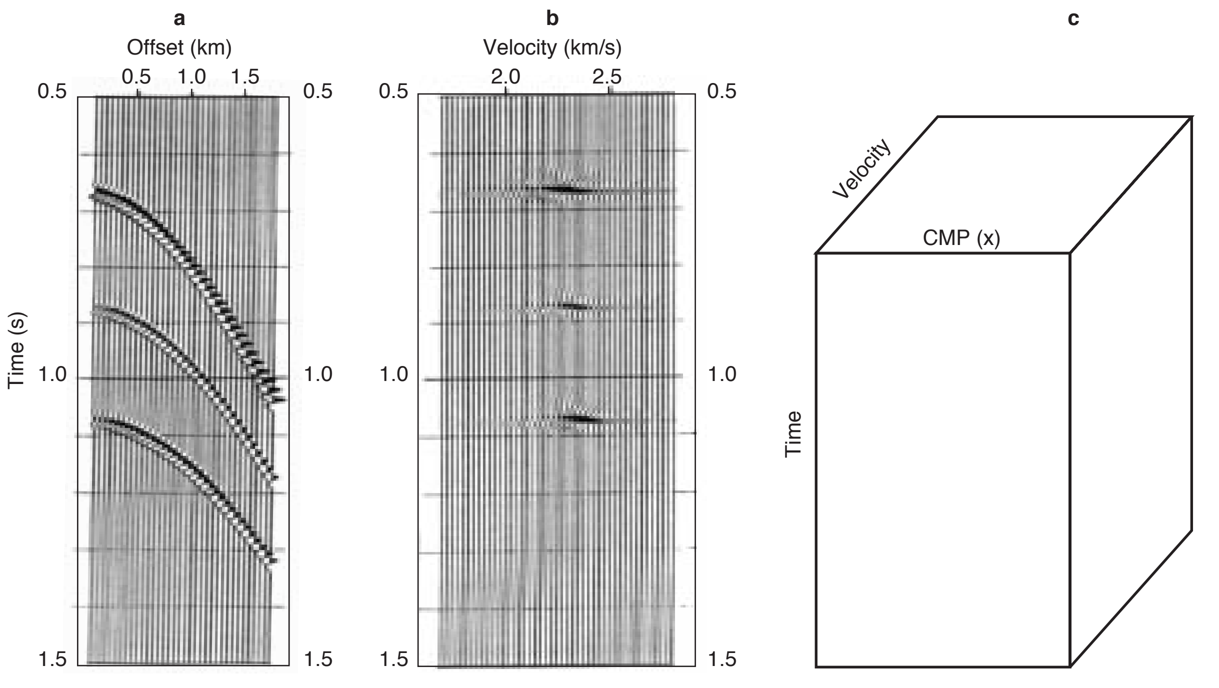

Figure 1

Illustration of how the migration-velocity analysis works.

(a) A CMP gather contains three reflection events.
(b) The result of scan over different velocity model. Notice that the maximal amplitudes correspond to the optimal velocity model.
(c) In practice, the results of scan are organized in to a 3D volume with vetical time, CMP and velocity as coordinates. We can then interactively, slide through the volume for a fixed CMP, to choose the best velocity model.

very popular and effective tool of seismic imaging, especially for migration-velocity analysis. Compared to other seismic prestack imaging techniques, the phase-shift migration is computationally fast, especially its F-K version which corresponds to homogeneous background. One reason why the isotropic prestack phase-shift migration is computationally efficient is that the dispersion relationship which controls the phase shift (downward continuation) operator is analytic. So, to extend the prestack phase-shift migration to anistropic media, we need analytic formulae of the dispersion relationship for anisotropic media.

In Ikelle (1996), I have derived analytic expressions of the dispersion relationship for anisotropic media. In this paper, I use those forumlae to produce a phase-shift migration for migration-velocity analysis. I will also use them to propose a cost-effective scanning procedure for TI and orthorhombic background media.

The formulation of anisotropic phase-shift migration described here is valid for an arbitrary distribution of sources and receivers as long as they are regularly sampled. However, I will use the common azimuthal geometry (i.e., a series of 2D multioffset profiles all oriented according to a particular azimuth) because this geometry allows me to reduce the amount of processing without compromising the resolution of the problem (Ikelle, 1995). In fact, the resolution (i.e., imaging plus AVO inversion) of elastic parameters for an azimuthally isotropic material at a given point in the subsurface depends on the frequency content and offset aperture of the data but not on the direction (azimuth) in which the offsets are acquired; this information is preserved by the organization of the data in common azimuthal section. For azimuthally anisotropic materials (e.g., orthorhombic media), I will consider more than one common azimuthal section.

In the next section, I review the formulae for the dispersion relation for anisotropic media. In the second section, I propose an anisotropic prestack phase-shift migration scheme. In the third section, I discuss its application to migration-velocity analysis.

1 DISPERSION RELATIONSHIP

One of the aims of this paper is to use the analytic formulae of dispersion relationships, based on weak anisotropy approximation, that I have derived in my previous work (Ikelle, 1996) to propose a prestack anisotropic phase-shift migration which is as computationally effective as the isotropic one. In this section, I review some of these formulae, essentially the ones that we need in the formulation of anisotropic phase-shift migration. For their derivation and validation, the reader is referred to Ikelle (1996).

If $\boldsymbol{k}_1$, $\boldsymbol{k}_2$are the horizontal wavenumbers corresponding to $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively, and if ω is the angular frequency corresponding to time $\boldsymbol{t}$, the relationship between the vertical wavenumber, denoted $\boldsymbol{q}$, and the angular frequency ω is called the dispersion relation. In the isotropic case for example, the dispersion relationship for *P*-wave is:

$$\boldsymbol{q}_0 = \frac{\omega}{\boldsymbol{V}}\sqrt{1 - \frac{\boldsymbol{V}^2\left[\boldsymbol{k}_1^2 + \boldsymbol{k}_2^2\right]}{\omega^2}} \tag{1}$$

Here $\boldsymbol{q}_0$ denotes the vertical wavenumber in the isotropic media; $\boldsymbol{q}$ is the reserved symbol for the vertical wavenumber in the anisotropic media. I will follow that convention through the remainder of the paper. Notice that the vertical wavenumber $\boldsymbol{q}_0$ is dependent on ω, $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$; that is also the case for $\boldsymbol{q}$. For short, I have omitted all these arguments.

Let us now look at how this formula generalizes for anistropic media. We start by examining two particular symmetries: transversely isotropic and orthorhombic symmetries.

1.1 Dispersion Relationship, Transversely Isotropic Case

For the case where the medium is transversely isotropic (TI) with respect to the vertical axis, the dispersion relationship can be written:

$$\boldsymbol{q} = \frac{\omega}{\boldsymbol{V_V}}\sqrt{1 - \frac{\boldsymbol{V}_{\boldsymbol{NMO}}^2\,\boldsymbol{k}^2}{\omega^2} - \alpha\frac{\boldsymbol{V}_{\boldsymbol{NMO}}^4\,\boldsymbol{k}^4}{\omega^4}} \tag{2}$$

where $\boldsymbol{V_V}$ is the vertical velocity, $\boldsymbol{V_{NMO}}$ is the normal move-out velocity, α is the anelliptic parameter and .

$$\boldsymbol{k} = \sqrt{\boldsymbol{k}_1^2 + \boldsymbol{k}_2^2}$$

We can remark that when $\alpha = 0$, (1) has an elliptical form:

$$\left[\frac{\boldsymbol{V_V}\,\boldsymbol{q}}{\omega}\right]^2 + \left[\frac{\boldsymbol{V_{NMO}}\,\boldsymbol{q}}{\omega}\right]^2 = 1 \tag{3}$$

Therefore α is the anelliptic parameter in (2). The ellipticity of this equation is described by the normal move-out velocity V_{NMO} and the vertical velocity V_V.

For processing in the time domain, the vertical wavenumber is replaced by a "vertical frequency" $\omega_\tau = V_V q$, which is explicitly defined as;

$$\omega_\tau = \omega \sqrt{1 - \frac{V_{NMO}^2 k^2}{\omega^2} - \alpha \frac{V_{NMO}^4 k^4}{\omega^4}} \qquad (4)$$

So, for TI processing of seismic data in the time domain, the dispersion relationship can be described by two parameters only: the normal move-out velocity V_{NMO} and the anellipticity α. Similar parametrization was obtained by Sayers (1994) and Alkhalifah *et al.* (1995).

1.2 Dispersion Relationship, Orthorhombic Case

For the case where the medium is orthorhombic, the dispersion relationship, through the vertical wavenumber q , can be written:

$$q = \frac{\omega}{V_V}\left[1 - \frac{V_{1,NMO}^2 k_1^2}{\omega^2} - \frac{V_{2,NMO}^2 k_2^2}{\omega^2} - \alpha_{12}\frac{V_{1,\mathrm{NMO}}^2 V_{2,NMO}^2 k_1^2 k_2^2}{\omega^4} - \alpha_{11}\frac{V_{1,NMO}^4 k_1^4}{\omega^4} - \alpha_{22}\frac{V_{2,NMO}^4 k_2^4}{\omega^4}\right]^{\frac{1}{2}} \qquad (5)$$

where V_V is the vertical velocity, V_{1NMO} and V_{2NMO} normal move-out velocities, and α_{11}, α_{12} and α_{22} are anelliptic parameters. Again, we remark that if $\alpha_1 = \alpha_{12} = \alpha_{22} = 0$, (5) describes an ellipsoid. It becomes:

$$\left[\frac{V_V q}{\omega}\right]^2 + \left[\frac{V_{1,NMO} k_1}{\omega}\right]^2 + \left[\frac{V_{2,NMO} k_2}{\omega}\right]^2 = 1 \qquad (6)$$

So, the anellipticity in the orthorhombic media is described by three parameters: α_{11}, α_{12} and α_{22}. The ellipsoidal behavior is described by a vertical velocity V_1 and two elliptical normal move-out velocities: $V_{1,NMO}$ and $V_{2,NMO}$.

For processing in the time domain, the vertical wavenumber is replaced by a "vertical frequency" $\omega_\tau = V_V\, q$, which is explicitly defined as:

$$\omega_\tau = \omega\left[1 - \frac{V_{1,NMO}^2 k_1^2}{\omega^2} - \frac{V_{2,NMO}^2 k_2^2}{\omega^2} - \alpha_{12}\frac{V_{1,NMO}^2 V_{2,NMO}^2 k_1^2 k_2^2}{\omega^4} - \alpha_{11}\frac{V_{1,NMO}^4 k_1^4}{\omega^4} - \alpha_{22}\frac{V_{2,NMO}^4 k_2^4}{\omega^4}\right]^{\frac{1}{2}} \qquad (7)$$

Notice that the vertical frequency ω_τ in Equation (7) can be described by three vertical frequencies corresponding to three TI media. In fact, if $k_2 = 0$, ω_τ, is reduced to a TI case, the normal move-out velocity and the anellipticity are $V_{1,NMO}$ and α_{11}, respectively. A similar remark can be made for $k_1 = 0$; the TI medium in this case is characterized by $V_{2,NMO}$ and α_{22}. If $k_1 = k_2$, the corresponding TI medium contains the last parameter, the anellipticity α_{12}, needed to completely reconstruct the orthorhombic velocity model.

2 FORMULATION OF ANISOTROPIC F-K MIGRATION

In the previous section, I introduced dispersion relationships. Here, I use these relationships to derive a prestack phase-shift migration.

The following formulation is divided in two parts. I start by defining the notion of common azimuthal section, the 3D acquisition geometry in this formulation. Then, I use the dispersion relationships in (1)-(7) to introduce anisotropic phase-shift migration.

2.1 Common Azimuthal Section

Consider a typical seismic reflection experiment. In what follows, (x_g, y_g, $z_g = 0$) denotes a generic receiver position and (x_s, y_s, $z_s = 0$) denotes a generic "shot" position. A typical seismic reflection data set can be represented by D (x_s, y_g, t; x_s, y_s). The time t is reset to zero at each shot.

When the background medium is considered homogeneous or depth dependent only as in the formulation of phase-shift migration, its practical implementation can be simplified by a change of variable from x_g, y_g, x_s, and y_s to the midpoint coordinates:

$$x = \frac{(x_g + x_s)}{2} \quad (8)$$

$$y = \frac{(y_g + y_s)}{2} \quad (9)$$

and half-offset coordinates

$$h = \frac{(x_g - x_s)}{2} \quad (10)$$

$$h' = \frac{(y_g - y_s)}{2} \quad (11)$$

The data $\boldsymbol{D}$ (x_g, y_g, t; x_s, y_s) can equivalently be represented by:

$$\boldsymbol{D}(x, y, h, h', t) = \boldsymbol{D}(x_g = x + h, y_g = y + h', t; x_s = x - h, y_s = y - h') \quad (12)$$

or its Fourier transformed version $\boldsymbol{D}$ (k_x, k_y, k_h, k'_h, ω). The variables k_x and k'_h are midpoint and half-offset wavenumbers which correspond to x, y, h, h' respectively. Notice that I have used the same symbol $\boldsymbol{D}$ with different arguments to express data in midpoint and half-offset, and its Fourier transform rather than defining a new variable. I will use this convention through the rest of the paper as the context will unambiguously indicate the quantity currently under consideration.

To facilitate the use of my anisotropic phase-shift migration scheme for the reconstruction of azimuthally anisotropic background, I have chosen to work with constant azimuthal sections instead of the full 3D seismic data. This is equivalent to taking one of the half-offset coordinates, say h', to be zero or h'/h to be constant. A common azimuthal section can be described as a series of 2-D multioffset profiles all oriented according to a particular azimuth. Ikelle (1995) has shown the saving in CPU time and memory gained from the use of constant azimuthal sections.

To fix our ideas, let us consider two examples of common azimuthal sections: $\boldsymbol{D}$ (k_x, k_y, k_h, h' = 0, ω) and $\boldsymbol{D}$ (k_x, k_y, h = 0, k'_h, ω). They represent 0° azimuthal section and 90° azimuthal section, respectively. To simplify notation, $\boldsymbol{D}$ (k_x, k_y, k_h, h' = 0, ω) will be renamed $\boldsymbol{D}$ (k_x, k_y, k_h, ω) and $\boldsymbol{D}$ (k_x, k_y, h = 0, $k_{h'}$, ω) will be renamed $\boldsymbol{D}'$ (k_x, k_y, k_h, ω).

Let us note that the inversion for this 0° azimuthal section can be used for any other constant azimuthal section because a non-0° azimuthal section is simply a rotation of the 0° azimuthal section with respect to the vertical axis; and a rotation of geographical frame is equivalent to replacing C_{ijpq} by a new C'_{ijpq} which is obtained as a linear combination of C_{ijpq}. Azimuthally isotropic media (e.g, TI media) are not affected by this rotation.

2.2 Phase-Shift Migration

Let $\omega_{\tau s}$ be the vertical frequency corresponding to downgoing wave from source to image point, $\omega_{\tau s}$ be the vertical frequency corresponding to upgoing wave from the image point to receiver, and $\boldsymbol{D}$ (k_x, k_y, k_h, ω) 3D data corresponding to a common azimuthal section, the phase-shift migration can be written:

$$\boldsymbol{M}(\boldsymbol{k}_x, \boldsymbol{k}_y, \tau) = \int_{-\infty}^{+\infty} \mathrm{d}\boldsymbol{k}_h \int_{-\infty}^{+\infty} \mathrm{d}\omega \, \exp\left[-\mathrm{i}\left(\omega_{\tau s} + \omega_{\tau g}\right)\tau\right] \boldsymbol{D}\left(\boldsymbol{k}_x, \boldsymbol{k}_y, \boldsymbol{k}_h, \omega\right) \quad (13)$$

with

$$\omega_{\tau s} = \omega_\tau \left(\boldsymbol{k}_1 = \frac{\boldsymbol{k}_x - \boldsymbol{k}_h}{2}, \ \boldsymbol{k}_2 = \frac{\boldsymbol{k}_y - \boldsymbol{k}_h^0}{2}, \omega\right) \quad (14)$$

$$\omega_{\tau g} = \omega_\tau \left(\boldsymbol{k}_1 = \frac{\boldsymbol{k}_x + \boldsymbol{k}_h}{2}, \ \boldsymbol{k}_2 = \frac{\boldsymbol{k}_y + \boldsymbol{k}_h^0}{2}, \omega\right) \quad (15)$$

and the wavenumber $\boldsymbol{k}_h^0$ is solution of:

$$\left. \frac{\partial \omega_\tau\left(\frac{\boldsymbol{k}_x - \boldsymbol{k}_h}{2}, \frac{\boldsymbol{k}_y - \boldsymbol{k}_h'}{2}, \omega\right)}{\partial \boldsymbol{k}_h'} + \frac{\partial \omega_\tau\left(\frac{\boldsymbol{k}_x + \boldsymbol{k}_h}{2}, \frac{\boldsymbol{k}_y + \boldsymbol{k}_h'}{2}, \omega\right)}{\partial \boldsymbol{k}_h'} \right|_{k_h' = k_h^0} = 0 \quad (16)$$

Equation (16) computes the half-offset wavenumber in the direction of the y axis. It shows that, although I organize data in parallel straight lines, lateral scattering and reflections in y-axis are naturally taken into account.

3 MIGRATION-VELOCITY ANALYSIS

3.1 Scanning Procedure: TI Media

Contrary to the isotropic case where the background medium is escribed by one parameter, it is described by several parameters in the anisotropic case. Therefore, for setting up a scanning procedure in the anisotropic case, it is important to scan over parameters that are as independent as possible, in terms of their relationship to travel times of seismic data, or over parameters that can be reconstructed hierarchically based on the significance of their contribution to travel times in seismic data. For reconstructing an azimuthally isotropic background velocity model (i.e., TI model), I have adopted a two-step scanning procedure. As the isotropic component is the dominant part for most anisotropic rock formations in the Earth, I propose to first scan over normal move-out velocities, $\boldsymbol{V_{NMO}}$, which characterize the travel times variations with offset in isotropic media. This scan is carried out over isotropic background velocity models using the phase-shift migration in Equation (13) with the dispersion relation in Equation (1). Once $\boldsymbol{V_{NMO}}$ is determined, a second scan over the anelliptic parameter α with the TI dispersion relationship (Eq. (4)) is carried out to adjust for anellipticity. So, the reconstruction of a TI velocity model requires two scans instead of one as in the isotropic case.

Let now use numerical examples to verify that this scanning procedure allows us to discriminate between different values of anellipticity.

Consider the geological model represented in Figure 2. The medium is composed of a homogeneous TI material overlying an isotropic material. The interface between the two materials is invariant with respect to the *y*-axis and contains dipping events. The elastic coefficients of this TI background are those of Bakken shale given in Vernik and Nur (1992). They are described in Table 1. Synthetic data composed of 198 gathers corresponding to midpoints spaced every 12 m, with offset coverage between 100 m to 5000 m were computed using the Born $2^{\frac{1}{2}}$ D forward modeling described by Ikelle (1995) with the exact vertical wavenumbers computed using the eigenvalue-eigenvector system described in Appendix A of Ikelle (1996).

The first scan is carried out over isotropic background characterized by $\boldsymbol{V_{NMO}}$ using an isotropic phase-shift migration. The velocities vary from 3600 to 4400 m/s with steps of 100 m/s. For clarity, I show only three migrated results (Fig. 3). By examining the amplitude of the horizontally flat events in these migrated results, we can see that $\boldsymbol{V_{NMO}}$ is about 4000 m/s. My selection of best velocity is based on the amplitudes of migrated results. Notice that an analysis of dipping events will lead to different values of $\boldsymbol{V_{NMO}}$. Therefore, there is no single isotropic constant velocity which can simultaneously migrate both horizontal and dipping events of this interface.

Let us now fix the normal move-out to 4050 m/s (which corresponds to optimal velocity for the horizontally flat events) and carry out the second scan over different values of anellipticity. Anellipticity is scanned from -0.1 to 0.3 with an increment of 0.05. Again, I show only three migrated results (Fig. 4). By examining the amplitude of these results, especially the dipping events, we can see that the best value of anellipticity is near 0.177. Therefore, we can

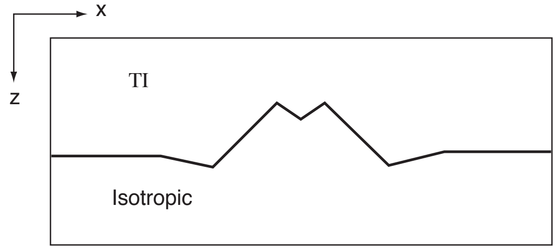

Figure 2

The model used to generate the data for the test of our scanning procedure. It consists of a homogeneous anisotropic medium above an isotropic one. The interface between these two media is invariant with respect to the *y*-axis and it contains four dips: 0°, 15°, 30°, and 45°.

TABLE 1

Density normalized stiffness tensor $\boldsymbol{a_{ijpq}}$ represented by its compact notation $\boldsymbol{A_{IJ}}$

Parameter	Value
$\boldsymbol{A}_{11}$	18.34
$\boldsymbol{A}_{33}$	12.06
$\boldsymbol{A}_{13}$	4.71
$\boldsymbol{A}_{44}$	4.71
$\boldsymbol{A}_{66}$	6.86

The medium here is TI. The $\boldsymbol{A_{IJ}}$ are expressed in $(km/s)^2$ and the velocities in km/s. The anelliptic parameters are dimensionless.

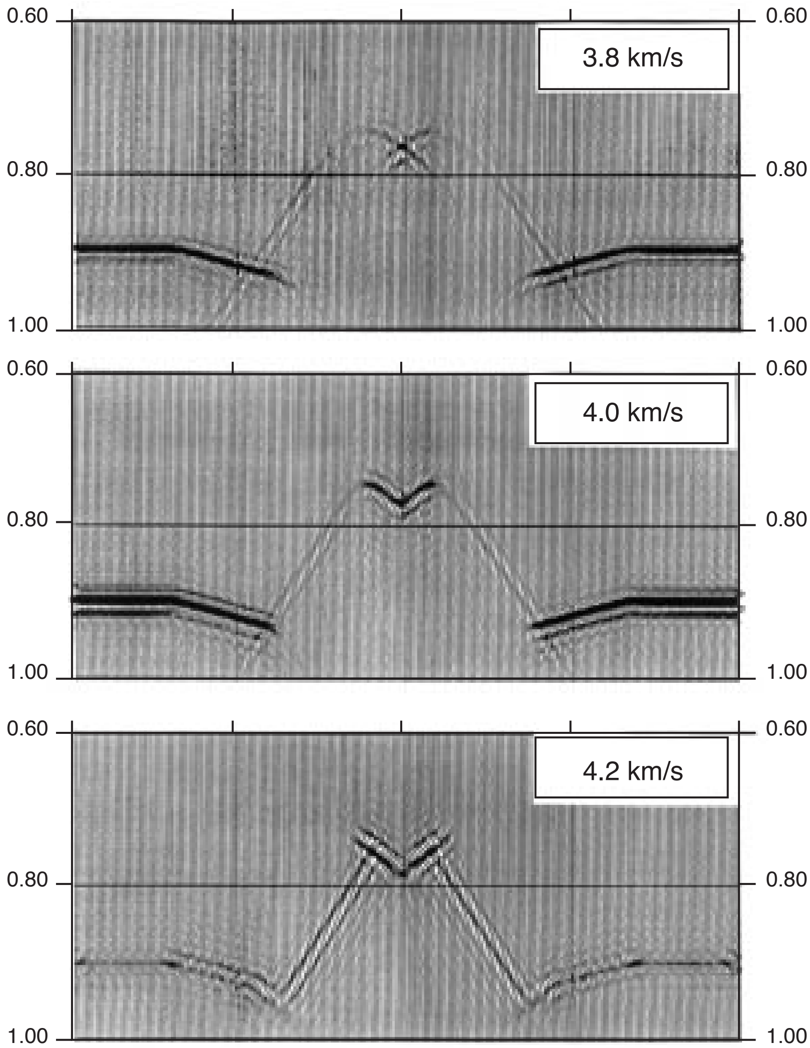

Figure 3

Scan over velocity V_{NMO} using phase-shift migration.

discriminate between different values of anellipticity. Furthermore, we can notice that one single TI velocity model (i.e., V_{NMO} and α) is now able to simultaneously migrate both horizontal and dipping events of this interface, just like depth migration.

I have also considered a real data example. The preprocessing consists of multiple attenuation and mute for direct and refracted waves. Figure 5 shows the migrated results corresponding to -0.05, 0.0 and +0.15 anellipticity with = 2000 m/s. Again, we can note improvement in the imaging of dipping reflectors with the variations of anellipticity. I have indicated by arrows one of these dipping reflectors.

3.2 Scanning Procedure: Orthorhombic Case

Since the dispersion relationship of orthorhombic media can be decomposed into TI dispersion relationships (Eq. (7)), we reconstruct the orthorhombic velocity model as three TI velocity models. As each TI medium in this decomposition was associated to a azimuthal angle, I found it useful to work with common azimuthal sections. For a given common azimuthal section, I use the two-step scanning procedure described earlier (i.e., we scan hierarchically over the normal move-out velocity and the anelliptic parameter) to reconstruct a TI velocity model.

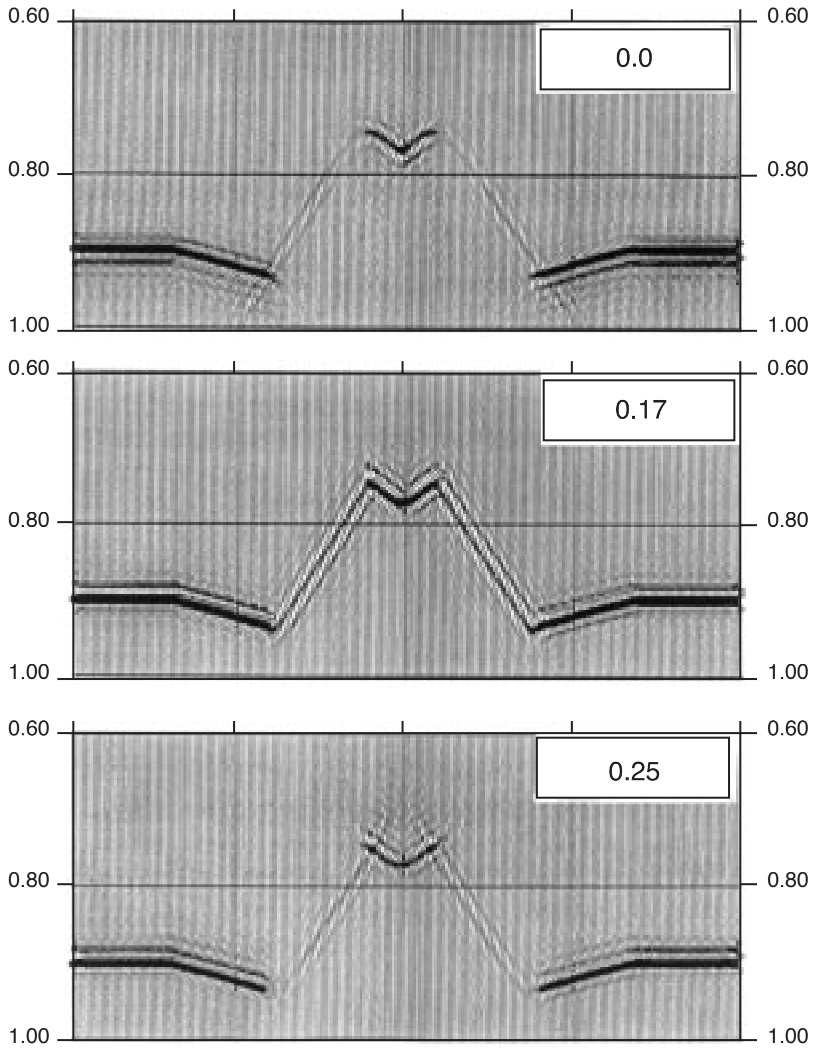

Figure 4

Scan over values of anellipticity with normal move-out velocity fixed to 4000 m/s.

Here is an example of implementation:

– I select a 0° azimuthal section. I then apply the two-step scanning procedure to reconstruct a TI velocity model as described earlier. The orthorhombic parameters reconstructed here are the normal move-out velocity $\boldsymbol{V}_{1,\boldsymbol{NMO}}$ and the anelliptic parameter α_{11}.
– I select a 90° azimuthal section. I then apply the two-step scanning procedure to reconstruct another TI velocity model. The orthorhombic parameters recovered here are the normal move-out velocity $\boldsymbol{V}_{2,\boldsymbol{NMO}}$ and the anelliptic parameter α_{22}.
– I select a 45° azimuthal section. I use the full orthorhombic dispersion relation (7) to adjust for the anelliptic parameter for α_{12}.

I have repeated the same numerical exercise carried out earlier by replacing the TI material in the model (Fig. 2) by an orthorhombic material. The elastic coefficients of the orthorhombic material are given in Table 2. Two common azimuthal sections, 0° azimuthal section and 90° azimuthal section, were computed. The 0° azimuthal section was formed as $\boldsymbol{D}(\boldsymbol{k}_x, \boldsymbol{k}_y, \boldsymbol{k}_h, \boldsymbol{h}' = 0, \omega)$, and 90° azimuthal section was formed as $\boldsymbol{D}$ $(\boldsymbol{k}_x, \boldsymbol{k}_y, \boldsymbol{h} = 0, \boldsymbol{k}_{h'}, \omega)$. As the medium is invariant with respect to the *y*-axis, $\boldsymbol{D}$ $(\boldsymbol{k}_x, \boldsymbol{k}_y, \boldsymbol{k}_h, \boldsymbol{h}' = 0, \omega)$ is equivalent to $\boldsymbol{D}$ $(\boldsymbol{k}_x, \boldsymbol{k}_y, \boldsymbol{k}_h, \boldsymbol{k}'_h = 0, \omega)$ and $\boldsymbol{D}$ $(\boldsymbol{k}_x, \boldsymbol{k}_y, \boldsymbol{h} = 0, \boldsymbol{k}_{h'}, \omega)$ is equivalent to $\boldsymbol{D}$ $(\boldsymbol{k}_x, \boldsymbol{k}_y, \boldsymbol{k}_h = 0, \boldsymbol{k}'_h, \omega)$.

For each common azimuthal section, 198 gathers corresponding to midpoints spaced every 12 m, with

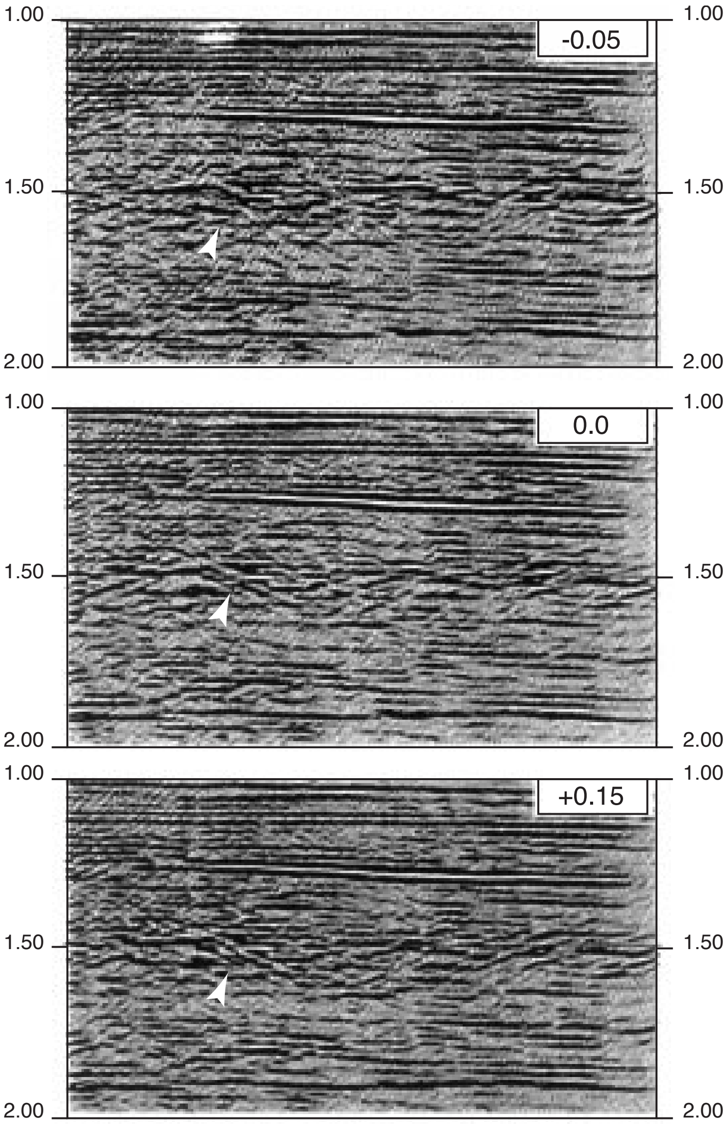

Figure 5

Scan over values of anellipticity with normal move-out velocity fixed to 2000 m/s using real data.

TABLE 2

Density normalized stiffness tensor $\boldsymbol{a}_{ijpq}$ represented by its compact notation $\boldsymbol{A}_{IJ}$

Parameter	Value
$\boldsymbol{A}_{11}$	12.79
$\boldsymbol{A}_{22}$	11.32
$\boldsymbol{A}_{33}$	8.56
$\boldsymbol{A}_{12}$	5.47
$\boldsymbol{A}_{13}$	5.14
$\boldsymbol{A}_{23}$	4.88
$\boldsymbol{A}_{44}$	2.30
$\boldsymbol{A}_{55}$	2.58
$\boldsymbol{A}_{66}$	2.76

The medium here is orthorhombic. The $\boldsymbol{A}_{IJ}$ are expressed in (km/s)^2 and the velocities in km/s. The anelliptic parameters are dimensionless.

offset coverage between 100 and 5000 m were computed using the Born forward modeling described by Ikelle (1995) with the exact vertical wavenumbers computed using the eigenvalue-eigenvector system described in Appendix A in Ikelle (1996).

We begin the process of background reconstruction with the 0° azimuthal section. The inversion is limited to 0° azimuthal angle (i.e., $\boldsymbol{D}$ ($\boldsymbol{k}_x$, $\boldsymbol{k}_y = 0$, $\boldsymbol{k}_h = 0$, $\boldsymbol{k}'_h$, ω)). As in the previous example, we first seek $\boldsymbol{V}_{1,NMO}$ by a scan over isotropic velocity models, and we then adjust for the anelliptic parameter α_{11} by a second scan. The scan over velocities is carried out for velocities between 2900 and 3700 m/s with an increment of 100 m/s.

The result, in Figure 6, shows that the best velocity is about 3500 m/s. The second scan is carried out over

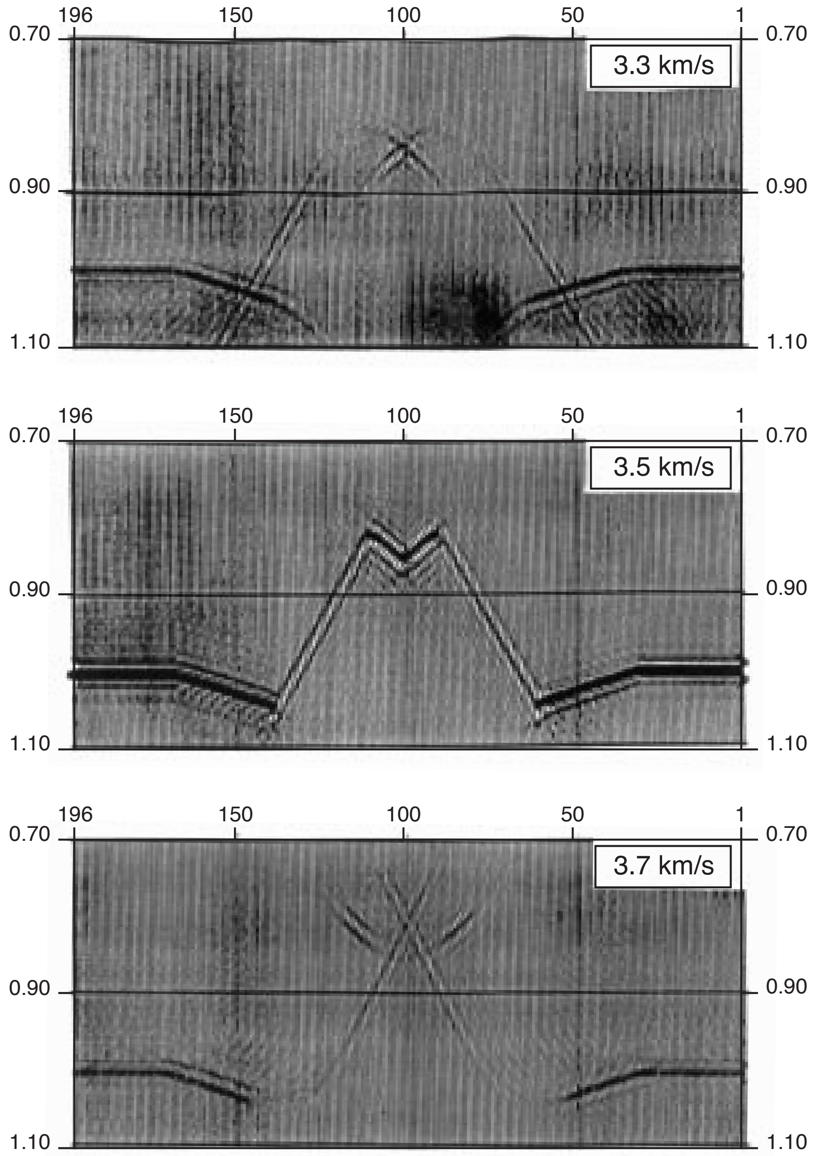

Figure 6

Scan over velocity $V_{1,NMO}$ (Eq. (7)). The background velocity model used to generate data is the orthorhombic medium described in Table 2. The data set here is a 0° azimuthal section.

different values of anellipticity with $V_{1,NMO}$ = 3500 m/s. The anellipticity varies from – 0.1 to 0.3 with an increment of 0.05. The results are shown in Figure 7. The best value of anellipticity, α_{11} in this case, is about 0.05.

I repeated the same exercise for the 90° azimuthal data. I estimated $V_{2,NMO}$ = 3200 m/s and α_{22} = 0.10.

To completely reconstruct the orthorhombic background velocity, I need to estimate the anelliptic parameter α_{12}. Using the 90° azimuthal data D (k_x, k_y, k_h = 0, k'_h, ω) with the full orthorhombic dispersion, I carry out one last scan over different values of the anelliptic parameter α_{12}. The values are taken between – 0.2 and 0.2 with an increment of 0.05 while the move-out velocities are fixed to $V_{1,NMO}$ =3500 m/s and $V_{2,NMO}$ = 3200 m/s, and the anelliptic parameters α_{11} and α_{22} are fixed to 0.05 and 0.10, respectively. The results are shown in Figure 8. The best value of anellipticity, α_{12} in this case, is about – 0.5. Again, we can notice a significant improvement in the imaging of dipping reflectors after correcting for the anellipticity compared to the isotropic results in Figure 6.

3.3 Velocity and Anelliptical Picking

In addition to scanning procedure, the other fundamental issue in migration-velocity analysis for anisotropic background media is the process of picking

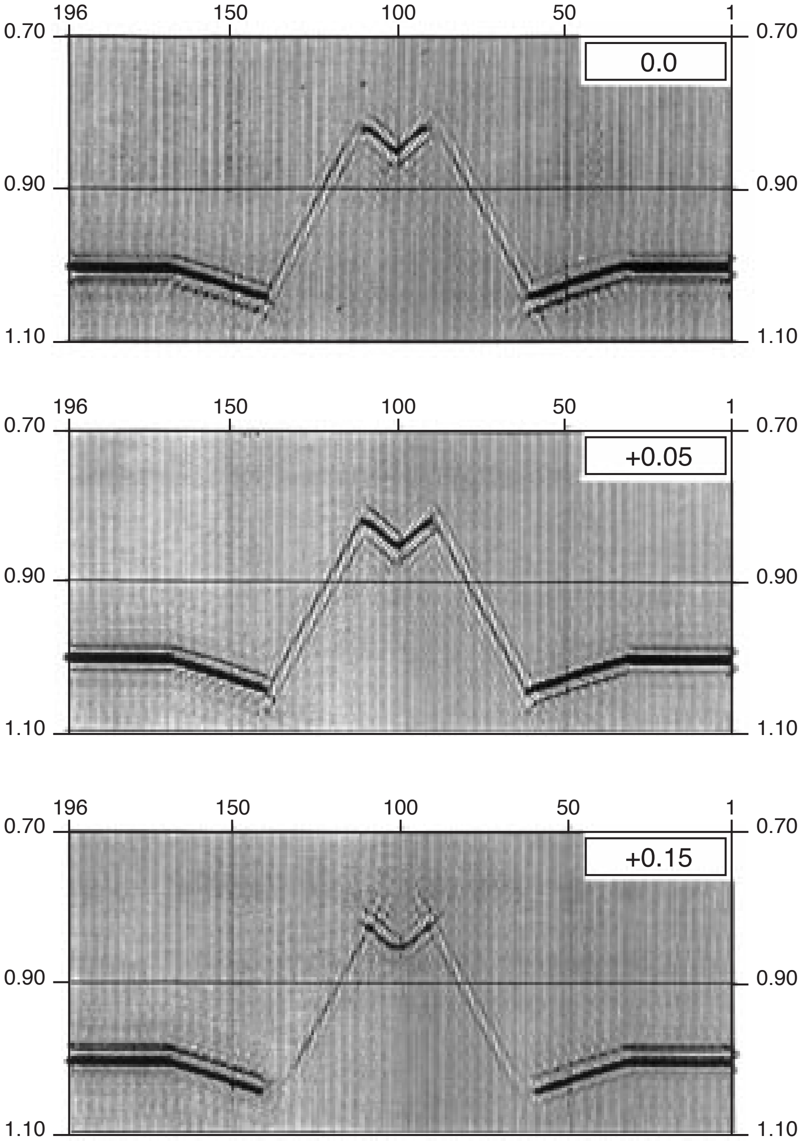

Figure 7

Scan over values of anellipticity, α_{11}, with normal move-out velocity fixed to 3500 m/s. The dispersion relationship used is TI. The background velocity model used to generate data is the orthorhombic medium described in Table 2. The data set here is a 0° azimuthal section.

the optimal values of anellipticy. To illustrate this problem, let us examine the number of occurrences of the vertical frequencies ω_τ in the phase-shift migration for TI media performed earlier.

Let us start by rewritting ω_τ as function of dip angles:

$$\omega_\tau = \omega\left\{\sqrt{1-\sin^2\theta_s - \alpha\sin^4\theta_s} + \sqrt{1-\sin^2\theta_g - \alpha\sin^4\theta_g}\right\} \tag{17}$$

where

$$\sin\theta_s = \frac{\boldsymbol{V_{NMO}}\left(\boldsymbol{k_x} - \boldsymbol{k_h}\right)}{2\omega} \tag{18}$$

and

$$\sin\theta_g = \frac{\boldsymbol{V_{NMO}}\left(\boldsymbol{k_x} + \boldsymbol{k_h}\right)}{2\omega} \tag{19}$$

For small angles θ_s and θ_g, the vertical frequency is essentially characterized by $\boldsymbol{V_{NMO}}$, while for large angles it is dominated by anellipticity α.

For a given survey, there is a large number of vertical frequencies which contribute to the migrated image. Figure 9 displays the number of occurrences as functions of dip angles θ_s and θ_g, for the TI numerical example described in Figures 2 and Table 1.

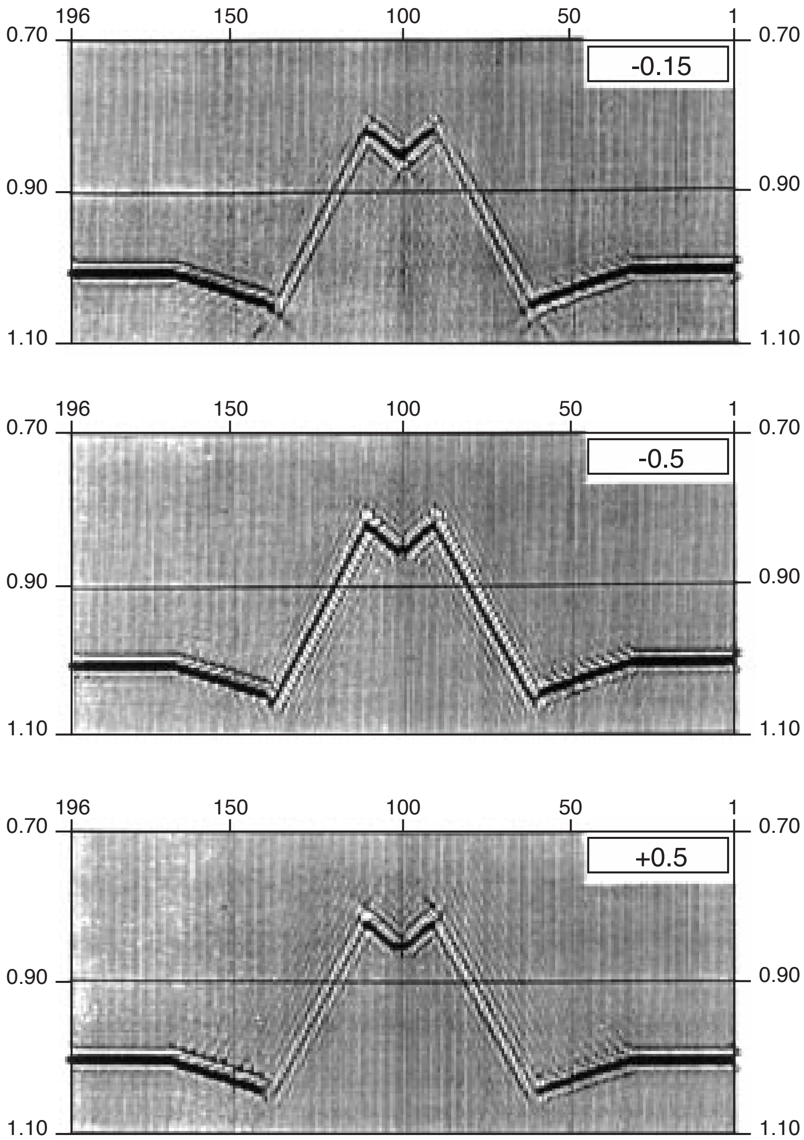

Figure 8

Scan over values of anellipticity, α_{12}. The inversion was carried out with the orthorhombic dispersion relationship. The move-out velocities were fixed to $\boldsymbol{V}_{1,NMO}$ = 3500 m/s and $\boldsymbol{V}_{2,NMO}$ = 3200 m/s, and the anelliptic parameters α_{11} and α_{22} were fixed to 0.05 and 0.10, respectively. The data set here is a 90° azimuthal section.

As the phase-shift migration is based on the $\boldsymbol{l}_2$ norm (this assumption is generally implicit in most migration formulations), it makes use of all vertical frequencies globally, and therefore the migrated section is dominated by the vertical frequencies that occur most frequently. As most occurrences are in the region where the TI medium behaves as an isotropic medium (see the distribution in Figure 9), the TI migrated results primarily characterize $\boldsymbol{V}_{NMO}$. That is why the picking of anellipticity is generally not precise enough if it is directly conducted on TI migrated results. This problem can be overcome by subtracting the results of isotropic migration from those of the TI migration before the picking process for the values of anellipticity, instead of using directly the TI migrated results.

Let me also remark that the scan of $\boldsymbol{V}_{NMO}$ must be carried out with small offset data to avoid picking the stack velocity instead of the normal move-out velocity.

CONCLUSIONS

This paper demonstrates that the problem of reconstructing an anisotropic background medium, in particular for TI and orthorhombic symmetries, can be solved by migration-velocity analysis in sequential scans:
– scans over normal move-out velocities first and
– scans over anellipticities second.

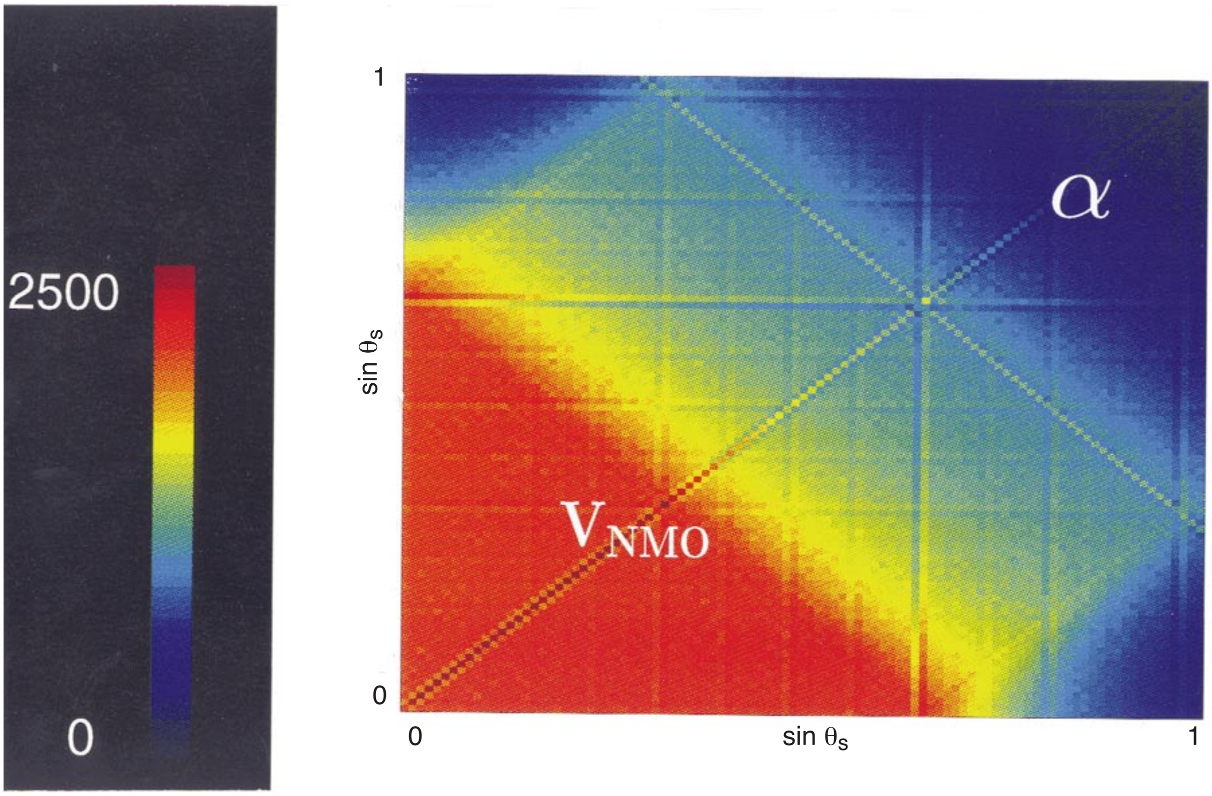

Figure 9

Histogram of vertical frequencies associated with the multi-offset synthetic data used in the TI migrations in Figure 4.

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