Multiple attenuation and P/S splitting of OBC data: A heterogeneous sea floor

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Summary

One of the benefits of dual-sensor (hydrophone and geophone) recordings in ocean bottom cable (OBC) experiments is that hydrophone data can be combined with geophone data to attenuate receiver-side reverberations in both data types and also for P/S splitting of geophone data albeit after applying appropriate scaling filters. Most of the formulae currently used to compute these scaling filters assume, sometimes implicitly, that the sea floor is horizontally flat. We present a generalization of these formulae to cases where the sea floor is arbitrary heterogeneous. Heterogeneities can be due to the topography of the sea floor as well as laterally varying medium parameters along the sea floor. The novelty here is that our derivations are based on the representation theorem instead of plane wave decomposition. If the medium parameters are locally constant along the receiver spread at a sea floor which need not be flat, the resulting scaling filters are explicit. As for the horizontally flat sea floor case, the computation of these scaling filters requires the knowledge of the acoustic impedance or velocities and density of the sea floor.

The results of our formulation can be used for multiple attenuation only or as a combination of multiple attenuation and P/S splitting.

Introduction

After several attempts through the **80's,** the technologies for recording seismic data directly on the sea floor are now well established. The expectations are that the combined use of pressure and shear seismic data will significantly improve, among others, lithology and fluid prediction, and in certain cases, seismic imaging. To fulfill these expectations, new seismic processing tools must be developed to accommodate the new OBC acquisition geometry. In particular the troublesome problem of multiple attenuation must be re-addressed. Also, it is of interest to decompose the vector recordings into scalar pressure and shear fields.

Two different classes of multiple suppression techniques for OBC data have been investigated. The first belongs to the class which aims at removing all free-surface related events in the seismic data (see, e.g., Matson and Weglein, **1996;** Ikelle, **1997).** The main advantage of this technique is that it does not require any knowledge of the subsurface. However, possible disadvantages are that it requires the source signature, it is computationally time-consuming, and it may not be easy to extend to demultiple of 3-D data.

The second class can be considered as an up/down splitting scheme just below the sea floor. Contrary to the free-surface multiple suppression technique, this latter technique does not remove the effect of the free-surface, but only receiver ghosts and its accompanying water layer reverberations (e.g., White, 1965; Barr and Sanders, 1989; Osen et al., 1997). The main advantages of this method are that it is computationally fast, and does not require information about the source signature. The main disadvantage is that it requires knowledge of the acoustic impedance or velocities and density of the sea floor. The current derivation of this latter technique has implicitly been based on assumptions of a horizontally flat sea floor.

We here describe a generalization of the second class of multiple suppression techniques. The new algorithm is not limited to the case of a horizontally flat sea floor; it may be arbitrarily heterogeneous. The heterogeneities can be due to the topography of the sea floor as well as laterally varying medium parameters along the sea floor. An example of practical interest is a dipping sea floor. In particular, if the sea floor medium parameters are locally constant over the receiver spread, simple analytical formulae for multiple suppression can be derived similar to those obtained for the case of a horizontally flat sea floor.

In addition to multiple attenuation, the preprocessing of OBC data sometime requires a P/S splitting of geophone data in particular. So, we also describe another version of this algorithm in which we combine multiple attenuation and P/S splitting.

Multiple attenuation: A heterogeneous sea floor with laterally varying medium parameters

The configuration of the problem under consideration is described in Figure **1.** The source that generates the wave motion is located in the water column while the receivers are located at the sea floor. The sea floor is considered heterogeneous; the heterogeneities can be due to the topography of the sea floor as well as laterally varying medium parameters of the elastic solid (just below the sea floor) on which the geophones are coupled. We will denote these medium parameters by $\alpha(\mathbf{x})$, $\beta(\mathbf{x})$ and $\rho(\mathbf{x})$ corresponding to P-wave velocity, S-wave velocity and density, respectively.

We have derived a demultiple scheme for this configuration. It is based on the elastodynamic representation theorem. To accommodate the occurrence of tensors, the subscript notation and the Einstein summation are adopted. Lower case Latin subscripts are employed for this purpose; they are to be assigned the values **1, 2** and 3. Lower case Greek subscripts are also employed; they are to be assigned the values **1** and **2.**

Let v_j be the jth component of the particle velocity, and p the pressure. The corresponding demultipled fields are denoted by $\tilde{v}_j$ and $\tilde{p}$. We have found that demultiple of

v_j and p amounts to evaluating the following integral

$$\tilde{v}_k(\mathbf{x},\omega) = v_k(\mathbf{x},\omega) - \oint_S dS(\mathbf{x}')\left\{p(\mathbf{x}',\omega)\left[i\omega G_{3k}(\mathbf{x},\mathbf{x}',\omega)\right]\right.$$

$$\left. -\ v_j(\mathbf{x}',\omega)\Sigma_{3jk}(\mathbf{x},\mathbf{x}',\omega)\right\}\ , \qquad (1)$$

and

$$\tilde{p}(\mathbf{x},\omega) = \frac{i\,\rho(\mathbf{x})}{\omega}\left\{\left[\alpha^2(\mathbf{x}) - 2\beta^2(\mathbf{x})\right]\partial_\mu\tilde{v}_\mu(\mathbf{x},\omega)\right.$$

$$\left. +\ \alpha^2(\mathbf{x})\partial_3\tilde{v}_3(\mathbf{x},\omega)\right\}\ , \qquad (2)$$

where S is the water/solid interface on which receivers are laid out (see Figure 1), G_{kl} and Σ_{ijk} are the elastodynamic Green's tensors pertaining to the elastic solid just below the sea floor $(\alpha(\mathbf{x}),\ \beta(\mathbf{x}),\ \rho(\mathbf{x}))$. G_{kl} is the Green's tensor correspondng to the particle velocity and Σ_{ijk} is the Green's tensor correspondng to the stress.

The inputs to the demultiple process based on equations (1) and (2) are the elastic parameters $(\alpha(\mathbf{x}),\ \beta(\mathbf{x}), \rho(\mathbf{x}))$ and a four-component OBC shot gather $(v_j,\ p)$. The output is another four-component shot gather $(\tilde{v}_j,\ \tilde{p})$.

Multiple attenuation: A heterogeneous sea floor with constant medium parameters

The configuration of the problem under consideration is described in Figure 2. The source that generates the wave motion is located in the water column while the receivers are located at the sea floor. The sea floor is considered heterogeneous but, contrary to the configuration in Figure **1,** the medium parameters are assumed constant. Actually, this assumption will hold in most cases.

As the medium parameters are constant, the Green's tensors G_{kl} and Σ_{ijk}, in Equation **(1),** are now explicit and shift invariant with respect to spatial coordinates $\mathbf{x}$. By using these two properties, we have reduced the demultiple of v_k and p *to*

$$\tilde{v}_\mu(\mathbf{x},\ \omega) = \frac{1}{2}\left\{v_\mu(\mathbf{x},\ \omega) + F_{v_3}^{(v_\mu)}(\mathbf{x},\omega) *\ v_3(\mathbf{x},\omega)\right\}\ ,$$

$$\tilde{v}_3(\mathbf{x},\omega) = \frac{1}{2}\left\{v_3(\mathbf{x},\omega) + F_{v_\mu}^{(v_3)}(\mathbf{x},\omega) * v_\mu(\mathbf{x},\omega)\right.$$

$$\left. +\ F_p^{(v_3)}(\mathbf{x},\ \omega) * p(\mathbf{x},\ \omega)\right\}, \qquad (3)$$

and

$$\tilde{p}(\mathbf{x},\ \omega) = \frac{1}{2}\left\{p(\mathbf{x},\ \omega) + F_{v_3}^{(p)}(\mathbf{x},\omega) * v_3(\mathbf{x},\ \omega)\right\}, \qquad (4)$$

where

$$F_{v_3}^{(v_\mu)}(\mathbf{x},\ \omega) = 2\ \Sigma_{33\mu}(\mathbf{x}, 0,\ \omega)\ ,$$

$$F_{v_\mu}^{(v_3)} = 2\ \Sigma_{3\mu 3}(\mathbf{x},\ 0,\ \omega)\ , \qquad (5)$$

$$F_p^{(v_3)}(\mathbf{x},\ \omega) = -2\ i\ \omega\ G_{33}(\mathbf{x},0,\ \omega)\ , \qquad (6)$$

$$F_{v_3}^{(p)}(\mathbf{x},\omega) = \frac{-2\,i\,\rho}{\omega}\left[\left(\alpha^2 - 2\beta^2\right)\partial_\mu\Sigma_{33\mu}(\mathbf{x},\ 0,\ \ \omega)\right.$$

$$\left. +\ \alpha^2\ \partial_3\Sigma_{333}(\mathbf{x},\ 0,\omega)\right]\ , \qquad (7)$$

where $*$ denotes spatial convolution. Notice that (3) combines geophone and hydrophone data, therefore additional filters for calibrating these two measurements may be needed in some practical situations. We refer to Osen et al. (1998) for examples of numerically optimized filters for multiple suppression.

Simulationeous multiple attenuation and P/S splitting

Some imaging techniques require that data be split into P- and S-waves, in addition to the application of demultiple. We here described another form of Equations **(1)** and (2) which allows us to combine demultiple with P/S splitting.

Amundsen and Reitan **(1995)** and Holvik et al. (1997) have presented decomposition filters, determined by plane-wave analysis, which extract the sea floor *P*- and S-wavefields from OBC data. More general filters, valid for a heterogeneous sea floor, are described here.

The configuration considered here is described in Figure **1.** Basically, the water/solid interface is aribitrary irregular and the elastic parameters of the solid on which the receivers are laid out, can vary laterally.

Let $\tilde{v}_j$ be the demultipled particle velocity. We can define the P/S splitting, through upgoing P-waves and *S*-waves potential fields, by applying the divergence and curl operator, respectively, to $\tilde{v}_j$, i.e.,

$$\tilde{\phi} = -i\omega\rho k_p^{-2}\partial_j\tilde{v}_j; \qquad \tilde{\psi}_j = i\omega\rho k_s^{-2}\epsilon_{jkl}\partial_k\tilde{v}_l; \qquad (8)$$

where ϵ_{jkl} is the Levi-Civita tensor (deHoop, 1995), $k_\alpha\ =\ \omega/\alpha$ **and** $k_\beta\ =\ \omega/\beta$ are the *P*- and *S*-**wave** wavenumbers, respectively, ϕ and ψ_k denote demultipled *P*- and S-waves potential fields. Substituting the expression of $\tilde{v}_j$ (Equation **(1))** in (8) leads to a new form of the demultiple of geophone data which describes both the demultiple and P/S splitting.

Let us look at the particular case where the water/solid interface is aribitrary irregular while the medium parameters of the solid are constant. This configuration is described in Figure 2. If we assume that evanescent waves have been filtered from the data, we can substitute the expression of $\tilde{v}_j$ in equation (3), which is more explicit, into (8), to arrive at the following solutions:

$$\tilde{\phi}(\mathbf{x},\ \omega) = \frac{1}{2}p(\mathbf{x},\ w) - i\,\omega\,\rho\,k_s^{-2}\ \partial_\mu v_\mu(\mathbf{x},\ \omega)$$

$$+F_{v_3}^{(\phi)}(\mathbf{x},\omega) * v_3(\mathbf{x},\omega)\,, \tag{9}$$

$$\tilde{\psi}_\mu(\mathbf{x},\omega) = F_p^{(\psi_\mu)}(\mathbf{x},\omega) * p(\mathbf{x},\omega)$$

$$+F_{v_j}^{(\psi_\mu)}(\mathbf{x},\omega) * v_j(\mathbf{x},\omega)\,, \tag{10}$$

$$\tilde{\psi}_3 = \partial_2 v_1(\mathbf{x},\omega) - \partial_1 v_2(\mathbf{x},\omega)\,, \tag{11}$$

where

$$F_{v_3}^{(\phi)}(\mathbf{x},\omega) = -i\omega\rho k_p^{-2}\partial_k \Sigma_{33k}(\mathbf{x},0,\omega)\,, \tag{12}$$

$$F_p^{(\psi_\mu)}(\mathbf{x},\omega) = \rho\beta^2 \epsilon_{\mu kl}\partial_k G_{3l}(\mathbf{x},0,\omega)\,, \tag{13}$$

$$F_{v_j}^{(\psi_\mu)}(\mathbf{x},\omega) = i\omega\rho k_s^{-2}\epsilon_{\mu kl}\partial_k \Sigma_{3jl}(\mathbf{x},0,\omega)\,, \tag{14}$$

where * denotes spatial convolution.

As for the demultiple algorithm described in Equation (1) and (2), the inputs to this second alogirthm, based on Equations (9), (10) and (11), are the elastic parameters (α, β, ρ) and a four-component OBC shot gather (v_j, p). However, the output is now the *P*- and S-waves potential fields $(\tilde{\phi}, \tilde{\psi}_j)$ instead of $(\tilde{p}, \tilde{v}_k)$. In other words, equations (8) to (14) combine multiple attenuation and P/S splitting.

Conclusions

We have described new formulae of OBC multiple attenuation and P/S splitting for the cases where the sea-floor is heterogeneous whether heterogeneities are due to the topography of the sea floor or laterally varying medium parameters. If the medium parameters are locally constant along the receiver spread at a sea floor which need not be flat, the resulting scaling filters are explicit.

We have also demonstrated that multiple attenuation and P/S splitting can be carried out simultaneously.

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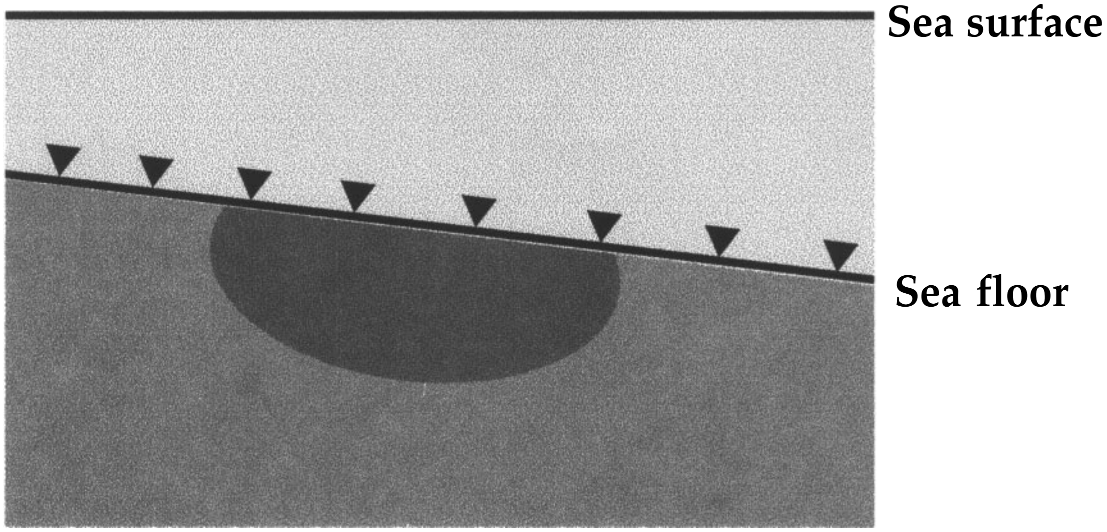

Figure 1: An example of heterogeneous sea floor with laterally variant medium parameters.

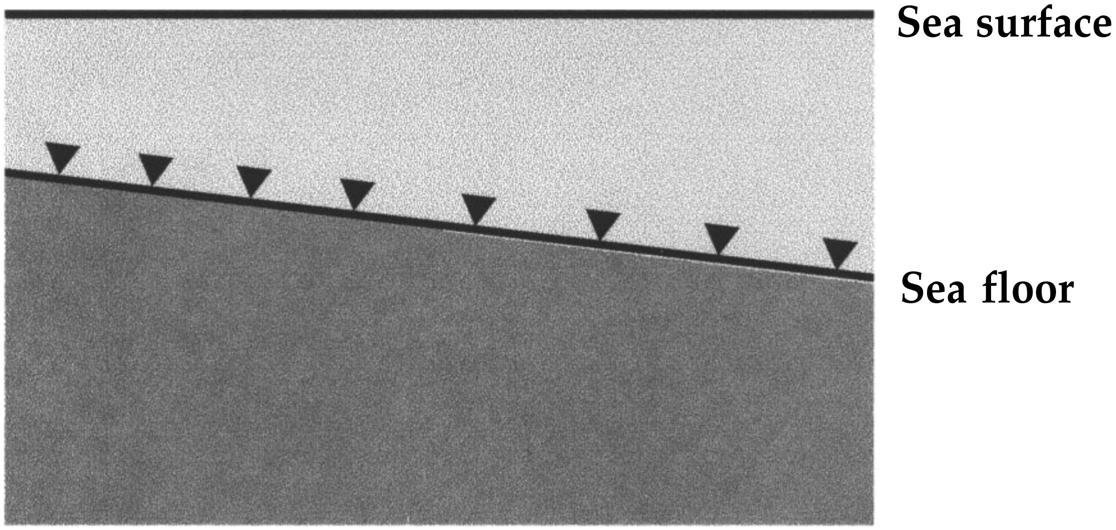

Figure 2: An example of heterogeneous sea floor with constant medium parameters.