GEOPHYSICS, VOL. 68, NO. 1 (JANUARY-FEBRUARY 2003); P. 16–28, 7 FIGS.
10.1190/1.1543191

Tutorial

Kirchhoff scattering series: Insight into the multiple attenuation method

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ABSTRACT

There are two basic integral equations used to represent wavefields in theoretical seismology: the Lippmann-Schwinger integral equation and the representation theorem. The Born scattering series currently used for attenuating free-surface multiples has been derived from the Lippmann-Schwinger integral equation. Similarly, we have used the representation theorem here to derive a Kirchhoff scattering series for attenuating free-surface multiples in towed-streamer data.

The Kirchhoff series for attenuating free-surface multiples is, in theory, equivalent to the Born series; most important, like the Born series, it does not require any knowledge of the subsurface. However, it still provides useful insight into the multiple-attenuation methods because the form of some quantities involved in the Kirchhoff series is different from the form in the Born series. For example, in towed-streamer seismic data, the Kirchhoff series requires measurements of the vertical derivative of the pressure field in addition to the pressure field itself, whereas the Born series requires only the pressure field and tries to obtain its vertical derivative by multiplying the pressure field in the *f-k* domain by the vertical wavenumber (generally known, in the context of multiple attenuation, as the obliquity factor).

The existing numerical implementations of the Born scattering multiple attenuation can be easily adapted to the Kirchhoff scattering multiple attenuation. We simply have to replace the computation of the vertical derivative of the pressure field with the recorded one. This is the most important conclusion in this paper for practitioners because they can use the numerically stable implementation of the inverse-scattering multiple-attenuation method in the form of a series when the combined measurements of the pressure field and its vertical derivative are available.

INTRODUCTION

When it became clear in the 1980s that 3D prestack imaging of seismic data could provide the accurate and detailed description of a reservoir needed for identification, evaluation, and even production of hydrocarbons, efforts to develop algorithms which allow us to fulfill the requirements of 3D prestack imaging intensified. Among these requirements were those related to the attenuation of multiple energy while preserving the characteristics of primaries (i.e., their amplitudes and phases). Through the 1990s, we saw developments of algorithms to fulfill this requirement (e.g., Verschuur et al., 1989, 1992; Fokkema and van den Berg, 1990, 1993; Carvalho et al., 1991, 1992; Verschuur, 1991; Dragoset and MacKay, 1993; Ikelle and Weglin, 1996; van Borselen et al.,1996; Ikelle and Jaeger, 1997; Ikelle, 1999a,b; Weglein et al., 1997, 1999; Berkhout, 1999; Ikelle et al., 2002). Furthermore, these algorithms do not require any knowledge of the subsurface. However, their cost and their prerequisites are still holding back their application in some situations. For instance, some of these algorithms, including the Born scattering multiple attenuation method, require knowledge of the source signature. Although solutions have been developed for this problem (e.g., Verschuur et al., 1992; Ikelle et al., 1997a), their usefulness is

Manuscript received by the Editor February 14, 2001; revised manuscript received May 15, 2002.
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sometimes limited by modeling errors such as the roughness of the sea surface, streamer feathering, and receiver ghosts, which are not taken into account in our predictions of multiples.

Amundsen (2001) and Amundsen et al. (2001) have proposed, for streamer data and for ocean-bottom seismic (OBS) data, respectively, an alternative way of removing free-surface multiples which does not have the same prerequisite as the Born scattering series solution. Whereas the Born scattering series solution is derived from the Lippmann-Schwinger integral equation, Amundsen et al.'s (2001) schemes are derived from the representation theorem. Assuming that pressure and its vertical derivative are both recorded during the seismic experiment, Amundsen et al. (2001) pose a solution which does not require knowledge of the source signature. However, their solution is derived as an integral equation, which may be difficult to implement for 2D and 3D media.

Starting from a representation theorem like Amundsen (2001) rather than from the Lippmann-Schwinger integral equation, our objective here is to derive, in a tutorial fashion, a Kirchhoff scattering series for attenuating free-surface multiples in seismic data, and to contrast it with the Born series.

Let us emphasize that our objective in this paper is not to describe a new algorithm but rather to present another derivation of the inverse scattering approach to multiple attenuation which may provide more insight to practitioners into the prediction and subtraction of multiples.

In the next section, we derive our Kirchhoff-series solution. In the third section, we rederive the Born series in a way that facilitates its comparison to the Kirchhoff series. In the last section, we contrast the two series and point out the advantages and drawbacks of each.

THE REPRESENTATION THEOREM AND THE KIRCHHOFF SCATTERING SERIES

Our objective in this section is to use the representation theorem to derive a Kirchhoff series for free-surface multiple removal. The basic idea is (1) to establish an integral relationship between the measured pressure field data containing all free-surface related multiples and data without free-surface multiples, and (2) to expand this integral relationship in the form of a series that we call the Kirchhoff series.

The representation theorem

We consider a 3D model of the earth consisting of an inhomogeneous solid half-space overlain by a homogeneous fluid (water) layer, as shown in Figure 1. The position in this configuration is specified by the coordinate $\mathbf{x} = (\chi, z)$, where $\chi = (x, y)$ represents the horizontal coordinates with respect to a fixed Cartesian reference frame with the origin at O and the three mutually perpendicular base vectors $\{\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3\}$. The unit vector $\mathbf{i}_3$ points vertically downward.

Before we introduce the representation theorem, we recall the wave equation that governs the recorded pressure field. If $p(\mathbf{x}, \omega; \mathbf{x}_s)$ denotes the Fourier transform of the pressure field for a receiver at $\mathbf{x}$ and a point source at $\mathbf{x}_s$, it obeys the following equation:

$$L(\mathbf{x}, \omega) p(\mathbf{x}, \omega; \mathbf{x}_s) = -s(\omega)\delta(\mathbf{x} - \mathbf{x}_s), \quad (1)$$

where

$$L(\mathbf{x}, \omega) = \omega^2 K(\mathbf{x}) + div\,[\sigma(\mathbf{x})\text{grad}], \quad (2)$$

with the condition that the pressure field vanishes at the free surface (i.e., at the sea surface); that is,

$$p(\chi, z = 0, \omega; \mathbf{x}_s) = 0, \quad (3)$$

where $K(\mathbf{x})$ is the compressibility (the reciprocal of the bulk modulus), $\sigma(\mathbf{x})$ is the specific volume (the reciprocal of density), and $s(\omega)$ is the source signature at point $\mathbf{x}_s$. We also introduce Green's function, which is associated with (1) and is denoted by $G(\mathbf{x}, \omega; \mathbf{x}')$, as follows:

$$L(\mathbf{x}, \omega) G(\mathbf{x}, \omega; \mathbf{x}') = -\delta(\mathbf{x} - \mathbf{x}'). \quad (4)$$

We do not specify the boundary conditions for Green's function because, in the representation theorem that we discuss later, we are free to choose the boundary condition which best suits our specific problem.

The representation theorem (Gangi, 1970; Aki and Richards, 1980) is expressed in terms of integrals over surfaces enclosing

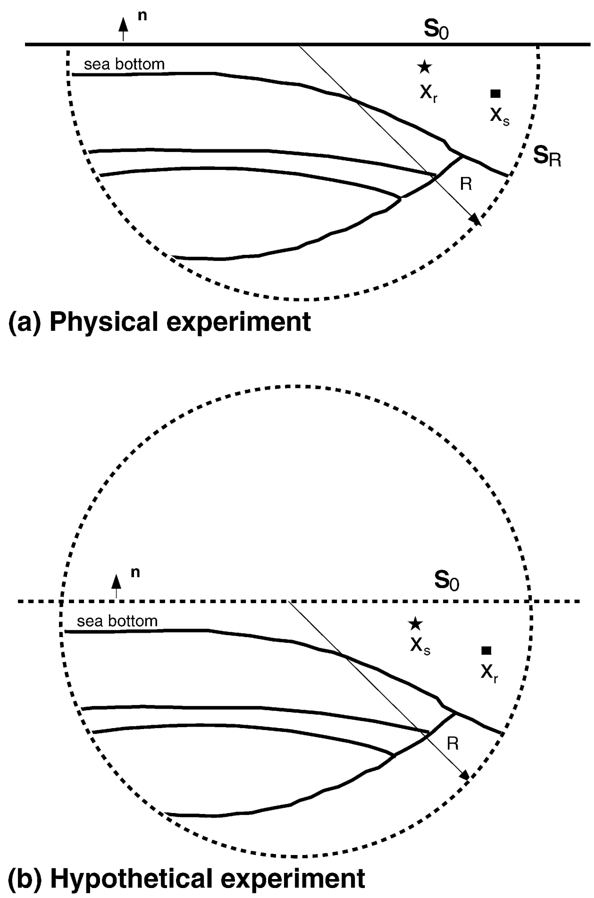

FIG. 1. Geometry of the physical and hypothetical seismic experiments. The surface $S = S_0 + S_R$ with an outward-pointing normal vector $\mathbf{n}$ encloses a volume V consisting of the water layer and the solid. (a) In the physical experiment, S_0 is a free surface with vanishing pressure. The source is positioned at a center location $\mathbf{x}_s$, and the receiver is located at $\mathbf{x}_r$. The free surface is a perfect reflector for all upgoing waves, which are reflected downward, giving rise to multiples. (b) In the hypothetical experiment, S_0 is a nonphysical boundary: all upgoing waves from the subsurface continue to propagate in the upward direction. No free-surface multiples are generated. The source is a monopole point source located at $\mathbf{x}_r$, and the receiver is located at $\mathbf{x}_s$. This figure is adapted from Amundsen (2001).

a volume. The question here is how to reconcile this property with our limited towed-streamer measurement along an open surface parallel to the sea surface. Our approach to this question is similar to that of Amundsen (2001). We consider a volume V enclosed by the surface $S = S_0 + S_R$, with an outward-pointing normal vector $\mathbf{n}$, as depicted in Figure 1, where S_0 is the air/water surface and S_R represents a hemisphere of radius R. The representation theorem solves for the pressure field inside volume V, assuming that the pressure on surface S, which bounds volume V, is known:

$$p(\mathbf{x}_r, \omega; \mathbf{x}_s) = G(\mathbf{x}_r, \omega; \mathbf{x}_s)s(\omega) + \oint_S dS(\mathbf{x})\sigma(\mathbf{x}) \left[G(\mathbf{x}, \omega; \mathbf{x}_r) \frac{\partial p(\mathbf{x}, \omega; \mathbf{x}_s)}{\partial n} - p(\mathbf{x}, \omega; \mathbf{x}_s) \frac{\partial G(\mathbf{x}, \omega; \mathbf{x}_r)}{\partial n} \right]. \quad (5)$$

The first term on the right side is included here because the sources are inside the volume. If we let radius R go to infinity, surface $S_{R\to\infty}$ gives a zero contribution to the surface integral in equation (5). This is Sommerfeld's radiation condition (Sommerfeld, 1954). Furthermore, using the boundary condition (3), equation (5) becomes

$$p(\mathbf{x}_r, \omega; \mathbf{x}_s) = G(\mathbf{x}_r, \omega; \mathbf{x}_s)s(\omega) + \sigma_0 \int_{S_0} dS(\chi) \times G(\chi, 0, \omega; \mathbf{x}_r) \frac{\partial p(\chi, 0, \omega; \mathbf{x}_s)}{\partial n}, \quad (6)$$

where $\sigma_0 = \sigma(\chi, 0)$ is the specific volume in the water. Using the fact that on S_0

$$\frac{\partial}{\partial n} = -\frac{\partial}{\partial z}, \quad (7)$$

and from the vertical component of the force-equilibrium equation, we have the following relationship between the vertical component of the particle velocity and the vertical derivative of the pressure field:

$$i\omega v_z(\mathbf{x}, \omega; \mathbf{x}_s) = \sigma_0 \frac{\partial p(\mathbf{x}, \omega; \mathbf{x}_s)}{\partial z}. \quad (8)$$

Hence, equation (6) can also be written as

$$p(\mathbf{x}, \omega; \mathbf{x}_s) = G(\mathbf{x}, \omega; \mathbf{x}_s)s(\omega) - i\omega \int_{S_0} dS(\chi) \times G(\chi, 0, \omega; \mathbf{x}_r) v_z(\chi, 0, \omega; \mathbf{x}_s). \quad (9)$$

Any of the Green's function of equation (4) can be used in equation (9). In other words, we are free to choose boundary conditions which suit our problem. So we have chosen a Green's function of an infinite medium which has the same 3D inhomogeneous solid half-space as that corresponding to the recorded data and which has an infinite water layer, as described in Figure 1b. We will denote it $G_P(\mathbf{x}, \omega; \mathbf{x}_r)$. Thus, the pressure field containing no free-surface multiples, source ghosts, or receiver ghosts can be written

$$p_P(\mathbf{x}_r, \omega; \mathbf{x}_s) = G_P(\mathbf{x}_r, \omega; \mathbf{x}_s)s(\omega), \quad (10)$$

where $p_P(\mathbf{x}_r, \omega; \mathbf{x}_s)$ denotes data without free-surface multiples and, hence, no receiver or source ghosts. Using equation (10), equation (9) becomes

$$p(\mathbf{x}_r, \omega; \mathbf{x}_s) = p_P(\mathbf{x}_r, \omega; \mathbf{x}_s) + a(\omega) \int_{S_0} dS(\chi) \times p_P(\chi, 0, \omega; \mathbf{x}_r) v_z(\chi, 0, \omega; \mathbf{x}_s), \quad (11)$$

or

$$G_a(\mathbf{x}_r, \omega; \mathbf{x}_s) = G_P(\mathbf{x}_r, \omega; \mathbf{x}_s) + \int_{S_0} dS(\chi) \times G_P(\chi, 0, \omega; \mathbf{x}_r) v'_z(\chi, 0, \omega; \mathbf{x}_s), \quad (12)$$

where

$$G_a(\mathbf{x}_r, \omega; \mathbf{x}_s) = \frac{p(\mathbf{x}_r, \omega; \mathbf{x}_s)}{s(\omega)}, \quad (13)$$

$$v_z{}'(\mathbf{x}, \omega; \mathbf{x}_s) = a(\omega) v_z(\mathbf{x}, \omega; \mathbf{x}_s), \quad (14)$$

and

$$a(\omega) = -\frac{i\omega}{s(\omega)}. \quad (15)$$

Equation (11) is the desired integral relationship between the pressure field without free-surface multiples $p_P(\mathbf{x}_r, \omega, \mathbf{x}_s)$ and the recorded data $p(\mathbf{x}_r, \omega, \mathbf{x}_s)$. Alternatively, we can use the integral relationship (12) between the deconvolved recorded data $G_a(\mathbf{x}_r, \omega; \mathbf{x}_s)$ and Green's function $G_P(\mathbf{x}, \omega, \mathbf{x}_s)$, which allows us to eliminate the source signature in the demultiple process, as we discuss in the fourth section. Therefore, we can use either integral equations (12) or (11) in our derivation of the Kirchhoff series. We opted to solve equation (11) simply to facilitate the comparison between the Kirchhoff and Born series that we will carry out later.

Let us now physically interpret equation (11), which relates seismic data containing primaries, free-surface multiples, and source and receiver ghosts to data which do contain these components. The first term on the right side contains primaries and internal multiples. As illustrated in Figure 2, the second term (which is a combination of p_P and v_z) predicts all free-surface multiples and receiver and source ghosts. Notice that fields p_P and v_z contain direct waves, allowing us to predict receiver and source ghosts of primaries.

Extrapolation of the vertical component of the particle velocity from the receiver positions to the sea surface

Before we discuss our solution of integral equation (11), let us remark that equation (11) requires v_z at the free surface, S_0. It is therefore necessary to extrapolate from the actual receiver point, (χ, z_r), to the point at the free surface, $(\chi, z = 0)$. Similarly, we needed to extrapolate the pressure field inside the integral from the source point at the free surface, $(\chi, z = 0)$, to the actual source point, (χ, z_s).

The particle-velocity field $v_z(\chi, z_r, \omega; \mathbf{x}_s)$ consists of an upgoing component, $u_v(\chi, z_r, \omega; \mathbf{x}_s)$, and a downgoing component, $d_v(\chi, z_r, \omega; \mathbf{x}_s)$. To get the particle-velocity field at the sea surface, we must forward-extrapolate the upgoing component from (χ, z_r) to $(\chi, z = 0)$ and backward-extrapolate the downgoing component from (χ, z_r) to $(\chi, z = 0)$. These two extrapolated fields must then be recombined to give the total particle-velocity field $v_z(\chi, z = 0, \omega; \mathbf{x}_s)$. Because of the free-surface boundary condition at the sea surface, we have

$$u_v(\chi, z=0, \omega; \mathbf{x}_s) = d_v(\chi, z=0, \omega; \mathbf{x}_s), \tag{16}$$

and since

$$v_z(\chi, z=0, \omega; \mathbf{x}_s) = u_v(\chi, z=0, \omega; \mathbf{x}_s) + d_v(\chi, z=0, \omega; \mathbf{x}_s), \tag{17}$$

we get

$$v_z(\chi, z=0, \omega; \mathbf{x}_s) = 2u_v(\chi, z=0, \omega; \mathbf{x}_s) = 2d_v(\chi, z=0, \omega; \mathbf{x}_s). \tag{18}$$

So we can get the total particle-velocity field at the sea surface by either forward-extrapolating the upgoing component or backward-extrapolating the downgoing component from the receiver location (χ, z_r) to $(\chi, z=0)$. We opted to forward-extrapolate the upgoing component.

Because we are assuming a dual measurement of a pressure field and its vertical derivative in this paper, we can use the Osen et al. (1999) formula to obtain the upgoing vertical particle velocity field, i.e.,

$$U_v(\boldsymbol{\kappa}, z_r, \omega; \mathbf{x}_s) = \frac{1}{2}\left[V_z(\boldsymbol{\kappa}, z_r, \omega; \mathbf{x}_s) - \sigma_0 \frac{k_z}{\omega} P(\boldsymbol{\kappa}, z_r, \omega; \mathbf{x}_s)\right], \tag{19}$$

with

$$k_z = \sqrt{\frac{\omega^2}{c^2} - k_x^2 - k_y^2}, \tag{20}$$

where $\boldsymbol{\kappa} = (k_x, k_y)$ represents the wavenumbers for the horizontal coordinates $\chi = (x, y)$ and where $V_z(\boldsymbol{\kappa}, z_r; \omega; \mathbf{x}_s)$ and $P(\boldsymbol{\kappa}, z_r, \omega; \mathbf{x}_s)$ are the 2D Fourier transforms of $v_z(\chi, z_r, \omega; \mathbf{x}_r)$ and $p(\chi, z_s, \omega; \mathbf{x}_r)$ with respect to χ, respectively. The quantity $U_v(\boldsymbol{\kappa}, z_r, \omega; \mathbf{x}_s)$ denotes the upgoing wavefield of the vertical particle velocity in the wavenumber domain.

The other field which occurs in the surface integral of equation (11) is the pressure $p_P(\chi, 0, \omega; \mathbf{x}_r)$ corresponding to the case of an infinite water layer. The desired pressure is $p_P(\chi, z_s, \omega; \mathbf{x}_r)$ instead of $p_P(\chi, 0, \omega; \mathbf{x}_r)$. To get the desired field, we need to extrapolate the field $p_P(\chi, z_s, \omega; \mathbf{x}_r)$ from the source location (χ, z_s) to $(\chi, z=0)$. As for the particle velocity, the pressure field $p_P(\chi, z_s, \omega; \mathbf{x}_r)$ consists of an upgoing component, $u_p(\chi, z_s, \omega; \mathbf{x}_r)$ and a downgoing component, $d_p(\chi, z_s, \omega; \mathbf{x}_r)$. To get the desired pressure at the sea surface, we have to forward-extrapolate the upgoing component and backward-extrapolate the downgoing component. However, here we have to distinguish between the case in which the source is located above the receiver and that in which the source is below the receiver. If the source lies above the receiver, then when the reciprocity theorem is invoked, i.e.,

$$p_P(\chi, z_s, \omega; \mathbf{x}_r) = p_P(\mathbf{x}_r, \omega; \chi, z_s) \tag{21}$$

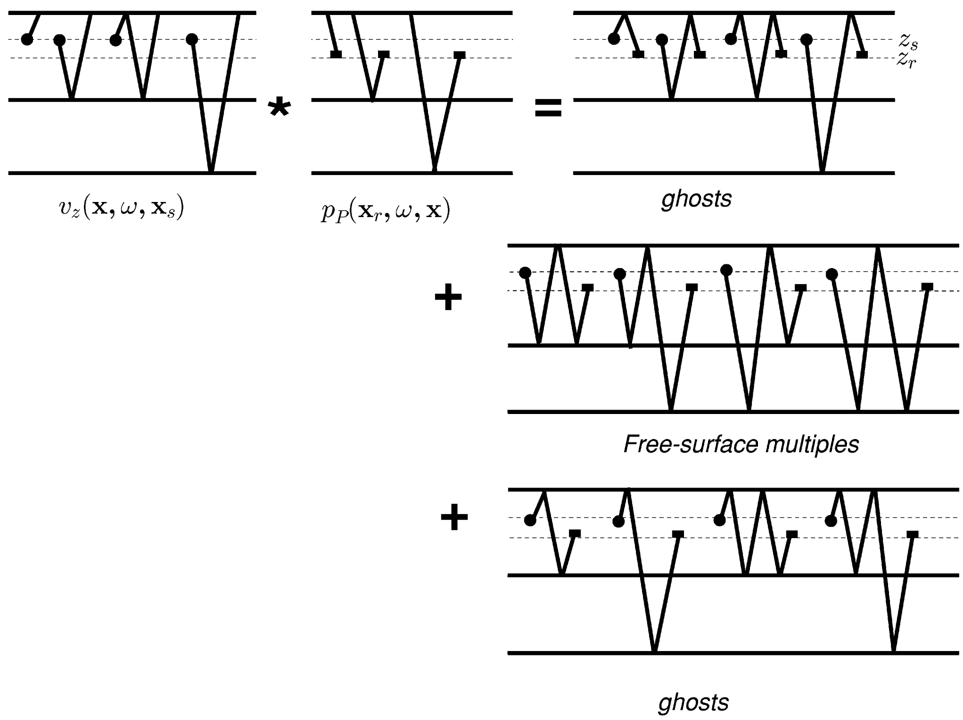

FIG. 2. Examples of the construction of free-surface multiples and source and receiver ghosts as a combination of pressure data, containing only primaries, and the vertical component of the particle velocity data. Symbols z_r and z_s are the depths of the receiver and shot points, respectively. The symbol * denotes the multidimensional convolution operations in the second term of equation (11), which allows us to combine v_z and p_P.

or

$$p_P(\chi, 0, \omega; \mathbf{x}_r) = p_P(\mathbf{x}_r, \omega; \chi, 0), \quad (22)$$

the result is a simulated source which lies below a simulated receiver. The complete desired pressure field at the simulated receiver, located at (χ, z_s), consists of the sum of the upgoing direct wave and the upgoing response from the subsurface. To get the complete desired pressure field at the level of the sea surface, $p_P(\chi, 0, \omega; \mathbf{x}_r)$, this sum of the upgoing direct wave and upgoing response from the subsurface must be forward-extrapolated from (χ, z_s) to $(\chi, z = 0)$. On the other hand, if the source lies below the receiver, then when reciprocity is invoked, the result is a simulated source which lies above a simulated receiver. The complete desired pressure field measured at the simulated receiver, located at (χ, z_s), consists of the sum of the downgoing direct wave and the upgoing subsurface response. To get the complete desired pressure field at the level of the sea surface for this case, the direct wave must be isolated and backward-propagated, and the subsurface response must be isolated and forward-extrapolated from (χ, z_s) to $(\chi, z = 0)$. The result must then be summed to get $p_P(\chi, 0, \omega; \mathbf{x}_r)$. The subsequent algebra is identical for both cases; i.e., the source is located above or below the receiver as long as the adequate extrapolation factors used. As sources are generally located above receivers in seismic acquisition, the derivations that follows are based on this case.

After some reorganization, equation (11) becomes

$$p(\mathbf{x}_r, \omega; \mathbf{x}_s) = p_P(\mathbf{x}_r, \omega; \mathbf{x}_s) + a(\omega) \int_{S_0} dS(\chi) \times p_P(\chi, z_s, \omega; \mathbf{x}_r) \tilde{v}_z(\chi, z_s, \omega; \mathbf{x}_s), \quad (23)$$

where

$$\tilde{v}_z(\chi, z_s, \omega; \mathbf{x}_s) = \int_{-\infty}^{+\infty} d\boldsymbol{\kappa}\, U_v(\boldsymbol{\kappa}, z_r, \omega, \mathbf{x}_s) \times \exp\{ik_z(z_s + z_r)\} \exp\{i\boldsymbol{\kappa}\chi\} \quad (24)$$

and where the term $\exp\{ik_z z_s\}$ is introduced by the extrapolation of the pressure field, p_P, inside the integral in equation (23) from the source point at the free surface $(\chi, z_s = 0)$ to the actual source point (χ, z_s), and the term $\exp\{ik_z z_r\}$ is introduced by the extrapolation of the vertical particle velocity field, U_z, from a receiver point (χ, z_r) to a point at the free surface. Figure 3 gives a physical illustration of field $\tilde{v}_z$ and the way it interacts with the field p_P to predict free-surface multiples and ghosts.

A Kirchhoff scattering series

Assuming that the recorded pressure field $p(\mathbf{x}_r, \omega; \mathbf{x}_s)$ and the recorded vertical component of the particle velocity $\tilde{v}_z(\chi, z_s, \omega; \mathbf{x}_r)$ are available, our next task is to construct the demultiple data, $p_P(\mathbf{x}, \omega; \mathbf{x}_s)$, by solving integral equation (23). We propose to solve this integral equation in the form of a series expansion that we call the Kirchhoff scattering series.

To construct the Kirchhoff scattering series, we start by rewriting equation (23) in the form

$$\int_{S_0} dS(\chi) \{I(\chi, \chi_s) + B_{\text{kir}}(\chi, z_s, \omega; \mathbf{x}_s)\} \times p_P(\mathbf{x}_r, \omega; \chi, z_s) = p(\mathbf{x}_r, \omega; \mathbf{x}_s), \quad (25)$$

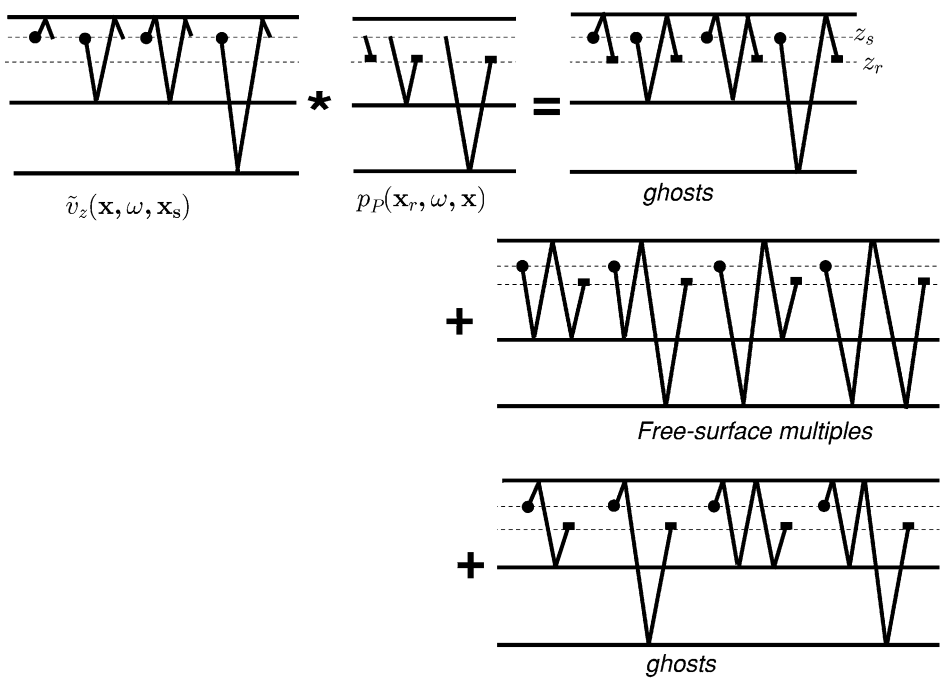

FIG. 3. An illustration of how the construction of free-surface multiples and source and receiver ghosts in Figure 2 are modified when using equation (23) instead of equation (11). Notice that the constructions of free-surface multiples and ghosts in Figures 2 and 3 yield the same events despite their differences.

where

$$B_{\text{kir}}(\chi, z_s, \omega; \mathbf{x}_s) = a(\omega)\tilde{v}_z(\chi, z_s, \omega; \mathbf{x}_s) \tag{26}$$

and

$$I(\chi, \chi_s) = \delta(\chi - \chi_s). \tag{27}$$

In compact notation, equation (25) becomes

$$[\mathbf{I} + \mathbf{B}_{\text{kir}}]\,\mathbf{p}_{\text{P}} = \mathbf{p} \tag{28}$$

or

$$\mathbf{p}_{\text{P}} = [\mathbf{I} + \mathbf{B}_{\text{kir}}]^{-1}\mathbf{p}. \tag{29}$$

The function $B_{\text{kir}}(\chi, z_s, \omega; \mathbf{x}_s)$ in equation (26) is the kernel of operator $\mathbf{B}_{\text{kir}}$, and $I(\chi, \chi_s)$ in equation (27) is the kernel of operator $\mathbf{I}$. By expanding equation (29) as a Taylor series, we arrive at the Kirchhoff scattering series:

$$\mathbf{p}_{\text{P}} = \left[\mathbf{I} - \mathbf{B}_{\text{kir}} + \mathbf{B}_{\text{kir}}^2 - \mathbf{B}_{\text{kir}}^3 + \ldots\right]\mathbf{p}, \tag{30}$$

which can be written as

$$\mathbf{p}_{\text{P}} = \mathbf{p} - \mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 + \ldots, \tag{31}$$

with

$$\mathbf{p}_{\text{n}} = \mathbf{B}_{\text{kir}}\,\mathbf{p}_{\text{n}-1}, \quad \text{n} = 1, 2, 3, \ldots. \tag{32}$$

A physical interpretation of the Kirchhoff scattering series is simple. The series is constructed using the recorded pressure $\mathbf{p}$ and its gradient, which is used to compute operator $\mathbf{B}_{\text{kir}}$. The first term of the series, $\mathbf{p}$, is the actual data; the second term, $\mathbf{p}_1$, removes events that bounce once at the free surface (i.e., receiver and source ghosts of primaries and first-order free-surface multiples). The next term, $\mathbf{p}_2$, removes events that bounce twice at the the free surface (i.e., receiver and source ghosts of first-order free-surface multiples, and second-order free-surface multiples), etc.

Explicitly, the Kirchhoff scattering series in equation (31), which removes free-surface multiples from 3D multioffset marine data, can be written as follows:

$$\begin{aligned} p_P(x_r, y_r, z_r, \omega; x_s, y_s, z_s) &= p(x_r, y_r, z_r, \omega; x_s, y_s, z_s) \\ &- p_1(x_r, y_r, z_r, \omega; x_s, y_s, z_s) \\ &+ p_2(x_r, y_r, z_r, \omega; x_s, y_s, z_s) - \ldots. \end{aligned} \tag{33}$$

The fields $\mathbf{p}_1$, $\mathbf{p}_2$, etc., are given by

$$\begin{aligned} p_n(x_r, y_r, z_r, \omega; x_s, y_s, z_s) &= a(\omega)\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \\ &\times p_{n-1}(x_r, y_r, z_r, \omega; x, y, z_s) \\ &\times \tilde{v}_z(x, y, z_s, \omega; x_s, y_s, z_s). \end{aligned} \tag{34}$$

The series in equation (33) follows the classical physical interpretation of a scattering series; i.e., the first term of the scattering series, p, is the actual data; the second term, p_1, aims at removing events which correspond to one bounce at the sea surface; the next term, p_2, aims at removing events which correspond to two bounces at the sea surface; and so on. Notice that events with bounces at the free surface include multiples as well as ghosts.

Let us elaborate on the physical interpretation of p_1, for instance. It is commonly assumed that the term p_1 contains only first-order multiples. This is not true; the term p_1 contains all orders of multiples. However, when it is scaled by the inverse of the source signature, a, it removes only the first-order multiples from the data because amplitudes of the higher orders of multiples contained in p_1 are not consistent. So we need the higher orders of series (33) to correct for this inconsistency in amplitude, which is essentially due to the fact that p_1 contains higher-order multiples several times instead of once, as described by Ikelle et al. (2002). Furthermore, p_1 contains some events that are not present in the actual data, as illustrated in Figure 4. Fortunately, adding the higher-order terms to p_1 cancels out these events.

Using the relationship in equation (8) between the vertical component of the particle velocity and pressure, the following Kirchhoff series for the particle velocity can be deduced from equation (33):

$$\begin{aligned} v_{z(P)}(x_r, y_r, z_r, \omega; x_s, y_s, z_s) &= v_{z(0)}(x_r, y_r, z_r, \omega; x_s, y_s, z_s) \\ &- v_{z(1)}(x_r, y_r, z_r, \omega; x_s, y_s, z_s) \\ &+ v_{z(2)}(x_r, y_r, z_r, \omega; x_s, y_s, z_s) - \ldots, \end{aligned} \tag{35}$$

with

$$\begin{aligned} v_{z(n)}(x_r, y_r, z_r, \omega; x_s, y_s, z_s) &= a(\omega)\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \\ &\times v_{z(n-1)}(x_r, y_r, z_r, \omega; x, y, z_s) \\ &+ \tilde{v}_z(x, y, z_s, \omega; x_s, y_s, z_s). \end{aligned} \tag{36}$$

where $v_{z(0)}$ is the actual particle velocity data and $v_{z(P)}$ is a field containing only primaries.

THE LIPPMANN-SCHWINGER EQUATION AND THE BORN SCATTERING SERIES

Derivations of the Born scattering series for removing free-surface multiples are well known (e.g., Carvalho et al., 1991; Ikelle and Weglein, 1996; Weglein et al., 1997; Ikelle and Amundsen, 2002). We have chosen to describe here the one in Ikelle and Amundsen (work in progress). Although the Ikelle and Amundsen derivations yield the same answer as the other existing derivations, they are the most suitable for a step-by-step comparison to those of the Kirchhoff scattering series presented earlier. For example, they include an integral equation similar to equation (23), which relates data with and without free-surface multiples (i.e., p and p_P), which is the starting point of our derivation of the Born scattering series.

Although we are not deriving the Born scattering series here from first principles as we did for the Kirchhoff scattering series, we still introduce the Lippmann-Schwinger integral equation from which the Born scattering series is derived. The key reason for doing so is that, in a later section, we use the Lippmann-Schwinger integral equation to point out that the Kirchhoff and Born series are actually equivalent.

The Lippmann-Schwinger equation for data with and without free-surface multiples

In accordance with the standard Born scattering description, we must now consider a reference medium in addition to an actual medium. In a towed-streamer configuration, the reference medium generally chosen is a homogeneous half-space containing water. We will go along with this choice in this section.

The pressure field $p_0 = p_0(\mathbf{x}, \omega, \mathbf{x}_s)$ corresponding to wave propagation in the reference medium satisfies the following equation:

$$L_0(\mathbf{x}, \omega) p_0(\mathbf{x}, \omega; \mathbf{x}_s) = -s(\omega)\delta(\mathbf{x} - \mathbf{x}_s), \qquad (37)$$

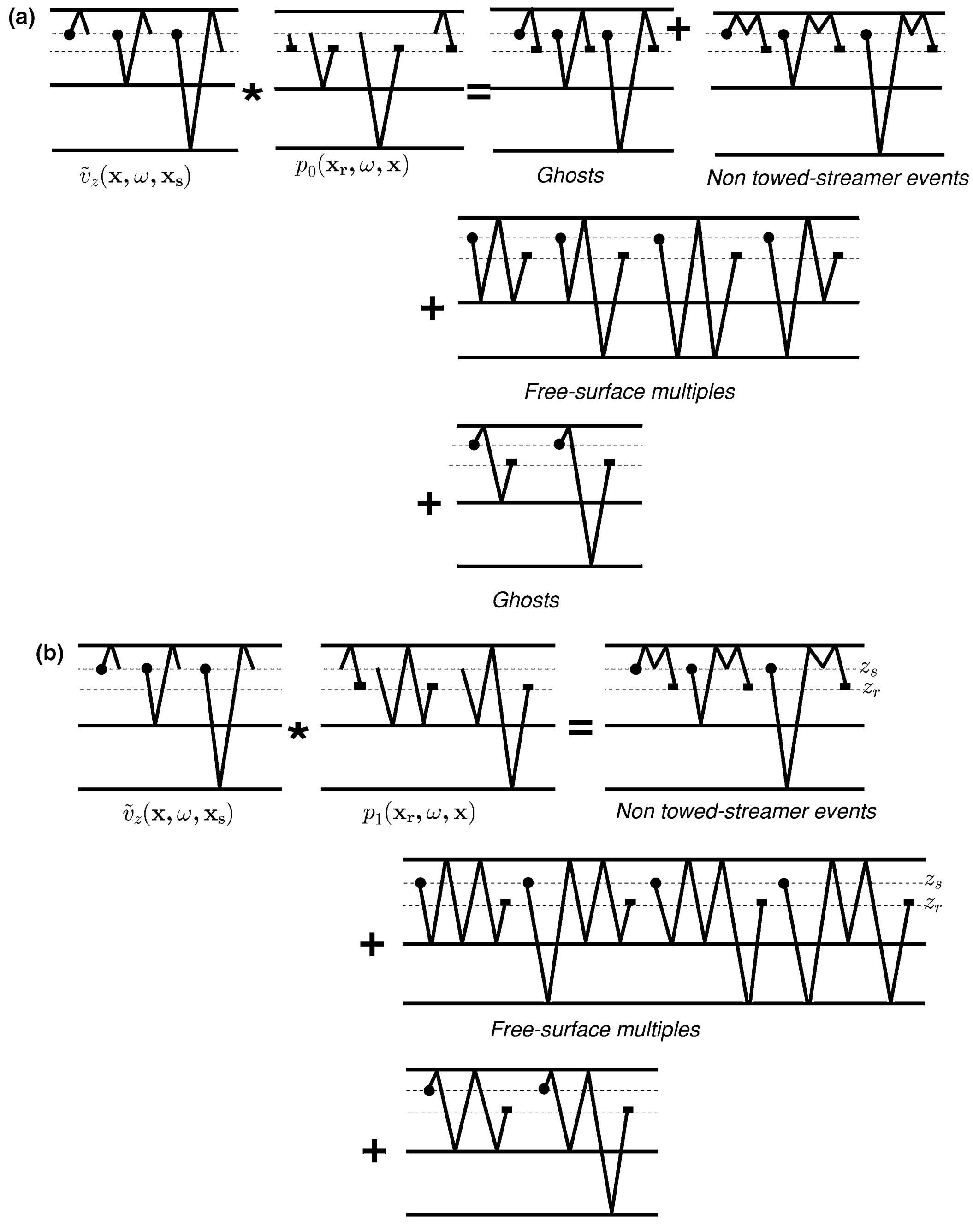

FIG. 4. (a) A combination of raw pressure data containing free-surface multiples and ghosts, and particle velocity to generate some of the events of the second term of the Kirchhoff series, p_1. (b) A combination of p_1 and particle velocity to generate some of the events of the third term of the Kirchhoff series, p_2. These combinations predict free-surface multiples as well as receiver and source ghosts. They also generate nontowed-streamer events which unfortunately cancel out as we add the higher order of the series. For example, the nontowed-streamer events in p_1 are also predicted by p_2. Therefore, these events cancel by adding p_1 and p_2, according to the Kirchhoff series in equation (34).

with

$$L_0(\mathbf{x},\omega) = \omega^2 K_0 + div\,[\sigma_0 \mathbf{grad}]\,. \quad (38)$$

$L_0 = L_0(\mathbf{x},\omega)$ is the differential operator describing wave propagation in the reference medium, K_0 is the compressibility (reciprocal of the bulk modulus), and σ_0 is the specific volume (reciprocal of density).

Later, we will also need Green's function, $G_0 = G_0(\mathbf{x},\omega,\mathbf{x}_s)$, which is associated with the wave equation for the reference medium. It is defined as follows:

$$L_0(\mathbf{x},\omega)G_0(\mathbf{x},\omega;\mathbf{x}') = -\delta(\mathbf{x}-\mathbf{x}'), \quad (39)$$

such that

$$p_0(\mathbf{x},\omega;\mathbf{x}_s) = s(\omega)G_0(\mathbf{x},\omega;\mathbf{x}_s). \quad (40)$$

Another quantity that we will also need later is the inverse of Green's function: $G_0^{-1} = G_0^{-1}(\mathbf{x},\omega;\mathbf{x}_s)$. It is defined as follows:

$$\int d\mathbf{x}' G_0(\mathbf{x},\omega;\mathbf{x}')G_0^{-1}(\mathbf{x}',\omega;\mathbf{x}'') = \delta(\mathbf{x}-\mathbf{x}''). \quad (41)$$

In addition to the reference medium, we are interested in the seismic response to two other media: the actual medium, which is associated with data containing primaries, free-surface multiples, and ghosts; and the hypothetical medium, which is associated with data containing primaries only. The hypothetical medium has the same inhomogeneous solid half-space as the actual medium but has an infinite water layer (see Figure 1). For the actual medium, pressure field $p = p(\mathbf{x},\omega;\mathbf{x}_s)$ satisfies the following equation:

$$L(\mathbf{x},\omega)p(\mathbf{x},\omega;\mathbf{x}_s) = s(\omega)\delta(\mathbf{x}-\mathbf{x}_s), \quad (42)$$

where $L = L(\mathbf{x},\omega)$ is the differential operator introduced in equation (2) for describing wave propagation in the actual medium.

The pressure field of primaries $p_P = p_P(\mathbf{x},\omega;\mathbf{x}_s)$ corresponding to wave propagation in the hypothetical medium described in Figure 1 satisfies the following equation:

$$L_P(\mathbf{x},\omega)p_P(\mathbf{x},\omega;\mathbf{x}_s) = s(\omega)\delta(\mathbf{x}-\mathbf{x}_s), \quad (43)$$

with

$$L_P(\mathbf{x},\omega) = \omega^2 K_P + div\,[\sigma_P \mathbf{grad}]\,. \quad (44)$$

$L_P = L_P(\mathbf{x},\omega)$ is the differential operator describing wave propagation in the hypothetical medium in Figure 1, K_P is the compressibility (reciprocal of the bulk modulus), and σ_P is the specific volume (reciprocal of density) of this hypothetical medium.

Using pressure field p_0 and Green's function G_0, the Lippmann-Schwinger equation gives us a solution for pressure field p, everywhere in the actual medium, via the following integral equation:

$$p(\mathbf{x},\omega;\mathbf{x}_s) = p_0(\mathbf{x},\omega;\mathbf{x}_s) + \int_D d\mathbf{x}' G_0(\mathbf{x},\omega;\mathbf{x}')V(\mathbf{x}',\omega)p(\mathbf{x}',\omega;\mathbf{x}_s), \quad (45)$$

where

$$V(\mathbf{x},\omega) = L(\mathbf{x},\omega) - L_0(\mathbf{x},\omega) \quad (46)$$

and domain D is the region of support of $V(\mathbf{x},\omega)$, as described in Figure 5. In compact notation, equation (45) becomes

$$\mathbf{p} = \mathbf{p}_0 + \mathbf{G}_0\mathbf{V}\mathbf{p}. \quad (47)$$

Function $G_0(\mathbf{x},\omega;\mathbf{x}')$ in equation (45) is the kernel of $\mathbf{G}_0$, and $V(\mathbf{x},\omega)$ in equation (46) is the kernel of the potential, $\mathbf{V}$.

The derivation of the Lippmann-Schwinger equation in equation (45) is given in Appendix A. You will notice that when this derivation is rigorously done, the Lippmann-Schwinger equation includes a surface integral over the surface bounding D in addition to the volume integral. Fortunately, in most seismic problems like the one considered here, this surface integral is null.

Using pressure field p_0 and Green's function G_0 again, the Lippmann-Schwinger equation can also be used to predict a solution for pressure field p_P corresponding to primaries only, everywhere in the actual medium, via the following integral equation:

$$p_P(\mathbf{x},\omega;\mathbf{x}_s) = p_0(\mathbf{x},\omega;\mathbf{x}_s) + \int_{D_P} d\mathbf{x}'\, G_0(\mathbf{x},\omega;\mathbf{x}') \times V_P(\mathbf{x}',\omega)\, p_P(\mathbf{x}',\omega;\mathbf{x}_s), \quad (48)$$

where

$$V_P(\mathbf{x},\omega) = L_P(\mathbf{x},\omega) - L_0(\mathbf{x},\omega) \quad (49)$$

and domain D_P is the region of support of $V_P(\mathbf{x},\omega)$, as described in Figure 5. Notice that the key difference between equations (45) and (48) is their scatterer potential. In compact notation, equation (48) becomes

$$\mathbf{p}_P = \mathbf{p}_0 + \mathbf{G}_0\mathbf{V}_P\mathbf{p}_P. \quad (50)$$

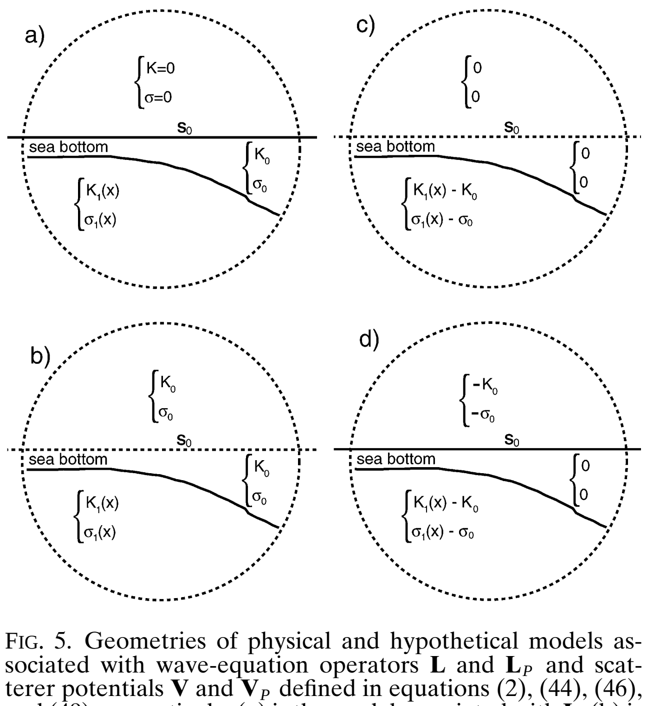

FIG. 5. Geometries of physical and hypothetical models associated with wave-equation operators $\mathbf{L}$ and $\mathbf{L}_P$ and scatterer potentials $\mathbf{V}$ and $\mathbf{V}_P$ defined in equations (2), (44), (46), and (49), respectively: (a) is the model associated with $\mathbf{L}$, (b) is the model associated with $\mathbf{L}_P$, (c) is the model associated with $\mathbf{V}$, and (d) is the model associated with $\mathbf{V}_P$.

Function $V_P(\mathbf{x}, \omega)$ in equation (49) is the kernel of the potential, $\mathbf{V_P}$.

Integral relationship between data with and without free-surface multiples

Ikelle and Amundsen (work in progress) have established a relationship between the actual data $\mathbf{p}$ and the data containing only primaries $\mathbf{p_P}$ which does not involve the scatterer potentials $\mathbf{V}$ or $\mathbf{V_P}$ like the one in equation (23) that we established earlier based on the representation theorem. This relationship can be written as follows:

$$[\mathbf{I} + \mathbf{B}_{\text{born}}]\mathbf{p}_{\text{P}}^{(\text{r})} = \mathbf{p}^{(\text{r})}, \tag{51}$$

with

$$\mathbf{B}_{\text{born}} = \mathbf{s}^{-1}\big[\mathbf{G}_{\text{P}}^{(0)}\big]^{-1}\mathbf{p}^{(\text{r})}, \tag{52}$$

where $\mathbf{p}^{(\text{r})}$ denotes the actual data without direct wave and without ghosts, $\mathbf{p}_{\text{P}}^{(\text{r})}$ denotes primaries without direct-wave, and $\mathbf{G}_{\text{P}}^{(0)}$ is the Green's function of an infinite homogeneous water space. Notice that the form of integral relationship (51) is similar to that of equation (28), with one major difference: equation (51) uses $\mathbf{p}^{(\text{r})}$ instead of $\mathbf{p}$. Therefore, the removal of the direct wave to obtain $\mathbf{p}^{(\text{r})}$ is now necessary. It also requires that the actual data be deghosted or that the effect of ghosts in the data be treated as part of an effective source signature.

The inverse Born scattering series

By expanding equation (51) as a Taylor series, we arrive at the inverse Born scattering series:

$$\mathbf{p}_{\text{P}}^{(\text{r})} = \big[\mathbf{I} - \mathbf{B}_{\text{born}} + \mathbf{B}_{\text{born}}^2 - \mathbf{B}_{\text{born}}^3 + \dots\big]\mathbf{p}^{(\text{r})}, \tag{53}$$

which can be equivalently written as

$$\mathbf{p}_{\text{P}}^{(\text{r})} = \mathbf{p}^{(\text{r})} - \mathbf{p}_1^{(\text{r})} + \mathbf{p}_2^{(\text{r})} - \mathbf{p}_3^{(\text{r})} + \dots, \tag{54}$$

with

$$\mathbf{p}_{\text{n}}^{(\text{r})} = \mathbf{B}_{\text{born}}\mathbf{p}_{\text{n-1}}^{(\text{r})}, \quad n = 1, 2, 3, \dots. \tag{55}$$

A physical interpretation of the inverse Born scattering series is similar to that of the inverse Kirchhoff scattering series. The series is constructed using the recorded pressure, $\mathbf{p}^{(\text{r})}$, the inverse source signature, $\mathbf{s}^{-1}$; and the inverse Green's function, $\mathbf{G}_{\text{P}}^{(0)}$, which is associated with the wave equation in the infinite homogeneous space. These three quantities are used in the computation of the operator $\mathbf{B}_{\text{born}}$. The first term of the series, $\mathbf{p}^{(\text{r})}$, is the actual data; the second term, $\mathbf{p}_1^{(\text{r})}$, aims at removing first-order free-surface multiples; the next term, $\mathbf{p}_2^{(\text{r})}$, aims at removing second-order free-surface multiples; and so on.

Explicitly, the inverse Born scattering series in equation (54) for removing free-surface multiples from 3D multioffset marine data can be written as

$$\begin{aligned} p_P^{(r)}(x_r, y_r, z_r, \omega; x_s, y_s, z_s) &= p^{(r)}(x_r, y_r, z_r, \omega; x_s, y_s, z_s) \\ &- p_1^{(r)}(x_r, y_r, z_r, \omega; x_s, y_s, z_s) \\ &+ p_2^{(r)}(x_r, y_r, z_r, \omega; x_s, y_s, z_s) + \dots, \end{aligned} \tag{56}$$

where $p_P^{(r)}(x_r, y_r, z_r, \omega; x_s, y_s, z_s)$ is the data without free-surface multiples, and the fields $p_1^{(r)}(x_r, y_r, z_r, \omega; x_s, y_s, z_s)$, $p_2^{(r)}(x_r, y_r, z_r, \omega; x_s, y_s, z_s)$, etc., are given by

$$\begin{aligned} p_n^{(r)}(x_r, y_r, z_r, \omega; x_s, y_s, z_s) &= \frac{1}{s(\omega)} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \\ &\times \hat{p}^{(r)}(x_r, y_r, z_r, \omega; x, y, z_s) \times p_{n-1}^{(r)}(x, y, z_s, \omega; x_s, y_s, z_s), \end{aligned} \tag{57}$$

with

$$\begin{aligned} \hat{p}^{(r)}(x_r, y_r, z_r, \omega; x, y, z_s) &= \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \left[G_P^{(0)}\right]^{-1} \\ &\times (x_r, y_r, z_r, \omega; x', y', z_s) \\ &\times p^{(r)}(x', y', z_s, \omega; x, y, z_s). \end{aligned} \tag{58}$$

Series (56) is currently used in practice by several scientists to demultiple seismic data. It assumes that data have been deghosted. Unfortunately, the deghosting operation without dual-field measurements is generally unstable, and it is rarely performed in practice prior to the demultiple.

CONTRASTING THE BORN AND KIRCHHOFF SERIES

Comparing equations (28) and (51)

As expected, the Kirchhoff series has the same structure as the Born series in the sense that its first term is the actual data, its second term aims at removing first-order free-surface multiples, its third term aims at removing second-order free-surface multiples, etc., just as in the Born series. However, they exhibit two major differences: (1) the Kirchhoff series uses the total pressure field, whereas the Born series assumes that the direct wave has been removed from the data; (2) the operator $\mathbf{B}_{\text{kir}}$, central to the derivation of the Kirchhoff series, is apparently different from $\mathbf{B}_{\text{born}}$.

The differences between the Born and Kirchhoff series can be traced to the two integral equations (28) and (51), which have led to series in equations (33) and (56), respectively. To understand these differences, we will establish the assumptions for passing from integral equation (28) to integral equation (51).

We start by decomposing the pressure, $\mathbf{p}$, the upgoing wavefield of the vertical particle velocity, $\tilde{\mathbf{v}}_{\mathbf{z}}$, and the field, $\mathbf{p}_{\text{P}}$, which represent the demultiple data, into an incident and reflected fields, as follows:

$$\mathbf{p} = \mathbf{p}^{(0)} + \mathbf{p}^{(\text{r})}, \tag{59}$$

$$\tilde{\mathbf{v}}_{\mathbf{z}} = \tilde{\mathbf{v}}_{\mathbf{z}}^{(0)} + \tilde{\mathbf{v}}_{\mathbf{z}}^{(\text{r})}, \tag{60}$$

and

$$\mathbf{p}_{\text{P}} = \mathbf{p}_{\text{P}}^{(0)} + \mathbf{p}_{\text{P}}^{(\text{r})}, \tag{61}$$

where $\{\mathbf{p}^{(0)}, \tilde{\mathbf{v}}_{\mathbf{z}}^{(0)}\}$ represent the incident field and $\{\mathbf{p}^{(\text{r})}, \tilde{\mathbf{v}}_{\mathbf{z}}^{(\text{r})}\}$ represent the reflected field. The field $\mathbf{p}_{\text{P}}^{(\text{r})}$ represents the demultiple data without the direct wave, and $\mathbf{p}_{\text{P}}^{(0)}$ represents the direct wave. In other words, $\mathbf{p}_{\text{P}}^{(0)}$ is the response of an infinite homogeneous space, whereas $\mathbf{p}^{(0)}$ is the response of a homogeneous half-space.

Just as we did for the total pressure field in the second section, the application of the procedure that leads to equation (23) can be adapted to the incident wave. Thus we arrive at

$$p^{(0)}(\mathbf{x}_r, \omega; \mathbf{x}_s) = p_P^{(0)}(\mathbf{x}_r, \omega; \mathbf{x}_s) + a(\omega) \int_{S_0} dS(\chi) p_P^{(0)} \times (\chi, z_s, \omega; \mathbf{x}_r) \tilde{v}_z^{(0)}(\chi, 0, \omega; \mathbf{x}_s) . \quad (62)$$

To facilitate our discussion, we will carry out the integral at $z_r = z_s = 0$. Substituting equations (59), (60), and (61) in equation (23) and using equation (62), we arrive at

$$\begin{aligned} p^{(r)}(\mathbf{x}_r, \omega; \mathbf{x}_s) = {} & p_P^{(r)}(\mathbf{x}_r, \omega; \mathbf{x}_s) + a(\omega) \int_{S_0} dS(\chi) p_P^{(0)} \\ & \times (\chi, 0, \omega; \mathbf{x}_r) \tilde{v}_z^{(r)}(\chi, 0, \omega; \mathbf{x}_s) \\ & + a(\omega) \int_{S_0} dS(\chi) p_P^{(r)}(\chi, 0, \omega; \mathbf{x}_r) \tilde{v}_z^{(0)} \\ & \times (\chi, 0, \omega; \mathbf{x}_s) + a(\omega) \int_{S_0} dS(\chi) p_P^{(r)} \\ & \times (\chi, 0, \omega; \mathbf{x}_r) \tilde{v}_z^{(r)}(\chi, 0, \omega; \mathbf{x}_s). \quad (63) \end{aligned}$$

As illustrated in Figure 6, the first term on the right side of equation (63) describes primaries, the second describes source ghosts of primaries, the third describes receiver ghosts of primaries and multiples, and the fourth describes free-surface multiples and their source ghosts. So to predict data containing primaries and multiples from the Kirchhoff series, we must assume that the vertical particle velocity does not contain direct-wave arrivals or source ghosts, in addition to our early requirement that the vertical particle velocity does not contain receiver ghosts. If we denote the receiver and source deghosted vertical particle velocity, in which direct-wave arrivals have been removed by $\tilde{\tilde{\mathbf{v}}}_{\mathbf{z}}^{(\mathrm{r})}$, then integral equation (63) of the reflected wavefield becomes

$$\begin{aligned} \tilde{\tilde{p}}^{(r)}(\mathbf{x}_r, \omega; \mathbf{x}_s) = {} & p_P^{(r)}(\mathbf{x}_r, \omega; \mathbf{x}_s) + a(\omega) \int_{S_0} dS(\chi) \\ & \times p_P^{(r)}(\chi, 0, \omega; \mathbf{x}_r) \tilde{\tilde{v}}_z^{(r)}(\chi, 0, \omega; \mathbf{x}_s), \quad (64) \end{aligned}$$

where $\tilde{\tilde{\mathbf{p}}}^{(\mathrm{r})}$ denotes pressure data without the direct wave and receiver and source ghosts; in other words, $\tilde{\tilde{\mathbf{p}}}^{(\mathrm{r})}$ contains primaries and free-surface multiples only.

Comparing equation (64) to equation (51), we can see that the difference between the integral equation leading to Kirchhoff and that leading to the Born series has been reduced to the difference between $\mathbf{B}_{\mathrm{kir}}$ and $\mathbf{B}_{\mathrm{born}}$:

$$B_{\mathrm{kir}}(x, y, 0, \omega; x_s, y_s, z_s) = a(\omega) \tilde{\tilde{v}}_z^{(r)}(x, y, 0, \omega; x_s, y_s, z_s), \quad (65)$$

$$\begin{aligned} B_{\mathrm{born}}(x, y, 0, \omega; x_s, y_s, z_s) = {} & \sigma_0 \frac{a(\omega)}{i\omega} \int_{-\infty}^{\infty} d\boldsymbol{\kappa} \\ & \times \left[G_P^{(0)}\right]^{-1}(x, y, 0, \omega; \boldsymbol{\kappa}, 0) P^{(r)}(\boldsymbol{\kappa}, 0, \omega; x_s, y_s, z_s), \quad (66) \end{aligned}$$

where $P^{(r)}(\boldsymbol{\kappa}, 0, \omega; x_s, y_s, z_s)$ is the 2D Fourier transform of $p^{(r)}(\chi, 0, \omega; x_s, y_s, z_s)$.

To understand the difference between $\mathbf{B}_{\mathrm{kir}}$ and $\mathbf{B}_{\mathrm{born}}$, it is useful to introduce the analytic expression of Green's function, $[G_P^{(0)}]^{-1}(x, y, 0, \omega, \boldsymbol{\kappa}, 0)$, which is involved in the computation of $\mathbf{B}_{\mathrm{born}}$. Using the expression of $[G_P^{(0)}]^{-1}(x, y, 0, \omega, \boldsymbol{\kappa}, 0)$ derived by Ikelle (1999b), we arrive at

$$\begin{aligned} & B_{\mathrm{born}}(x, y, 0, \omega; x_s, y_s, z_s) \\ & \quad = \sigma_0 \frac{a(\omega)}{i\omega} \int_{-\infty}^{\infty} d\boldsymbol{\kappa}\, i k_z P^{(r)}(\boldsymbol{\kappa}, 0, \omega; x, y, z_s) \exp\{-i\boldsymbol{\kappa}\chi\} \quad (67) \end{aligned}$$

or

$$\begin{aligned} B_{\mathrm{born}}(x, y, 0, \omega; x_s, y_s, z_s) = {} & a(\omega) \\ & \times v''_{zr}(x, y, 0, \omega; x_s, y_s, z_s), \quad (68) \end{aligned}$$

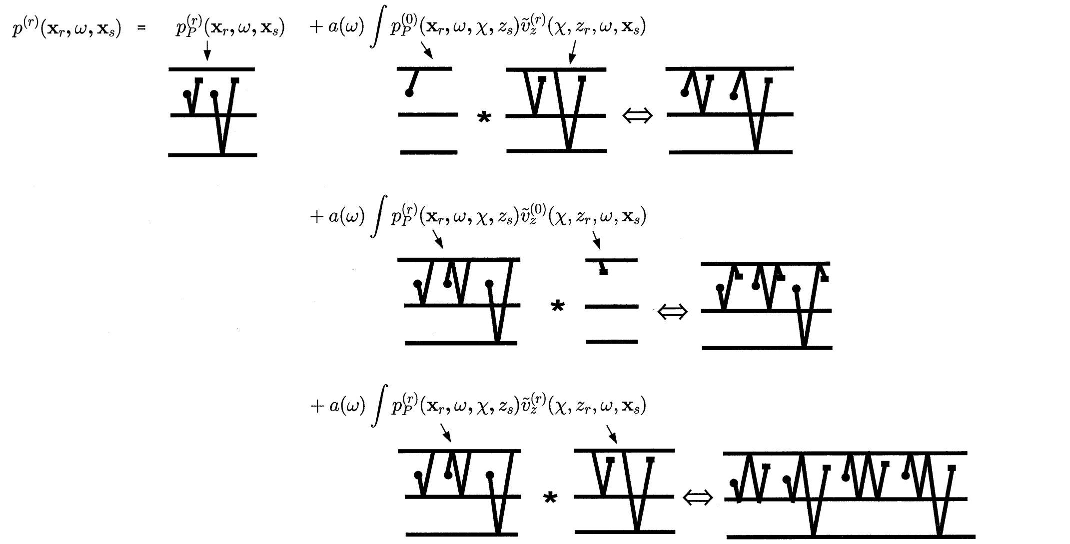

FIG. 6. A physical interpretation of the different terms of equation (63). The integrals are carried out at the sea surface (i.e., $z_r = z_s = 0$).

where

$$v''_{zr}(\chi, 0, \omega; \mathbf{x}_s) = \int_{-\infty}^{+\infty} d\boldsymbol{\kappa}\, U''_v(\boldsymbol{\kappa}, 0, \omega; \mathbf{x}_s) \exp\{i\boldsymbol{\kappa}\chi\} \tag{69}$$

and

$$U''_v(\boldsymbol{\kappa}, 0, \omega; \mathbf{x}_s) = \sigma_0 \frac{k_z}{\omega} P^{(r)}(\boldsymbol{\kappa}, 0, \omega; \mathbf{x}_s). \tag{70}$$

Notice that formulas (68) and (24) are similar. Both represent the vertical component of the particle velocity, and they are equivalent if the pressure data, $\mathbf{p}^{(\mathrm{r})}$, does not contain receiver and source ghosts. They differ in the way the particle velocity is obtained. In $\mathbf{B}_{\mathrm{born}}$, the particle velocity is computed from the pressure field, whereas in $\mathbf{B}_{\mathrm{kir}}$ we have assumed that it is recorded along with the pressure field. For this reason, we expect the Kirchhoff series to be less sensitive to the roughness of the sea surface and the effects of streamer feathering compared to the Born series.

The difference between $\mathbf{B}_{\mathrm{born}}$ and $\mathbf{B}_{\mathrm{kir}}$ also shows the importance of wavenumber k_z, which is generally known as the obliquity factor in multiple attenuation jargon. Ignoring the obliquity factor, as is often the case in some numerical implementations, is equivalent to replacing the vertical component of the particle velocity with the pressure field. Therefore, the obliquity factor should not be neglected.

In summary, we have established that the Born series is just a particular case of the Kirchhoff series if we assume that direct-wave arrivals and ghost effects have been removed from the data. The only difference between the two approaches is the way the vertical component of the particle velocity is obtained. It is numerically computed from the pressure in the Born series, whereas in the Kirchhoff series, we assume that it is recorded along the pressure field.

Why are there any differences at all between the Kirchhoff and Born series?

We have just pointed out some differences between the Kirchhoff and Born series. The next question is: why are there any differences at all between the Kirchhoff and Born series when the Lippmann-Schwinger integral equation is equivalent to the representation theorem? To address this question, let us first establish the equivalence between the Lippmann-Schwinger integral equation and the representation theorem.

If, instead of the homogeneous half-space introduced earlier, we consider the hypothetical medium without a free-surface, which is associated with the field of primaries, as the reference medium, then the actual field can be written

$$p(\mathbf{x}_r, \omega; \mathbf{x}_s) = p_P(\mathbf{x}_r, \omega; \mathbf{x}_s) + \int_{\Delta D} d\mathbf{x} \times G_P(\mathbf{x}_r, \omega; \mathbf{x})\, \Delta V(\mathbf{x}, \omega)\, p(\mathbf{x}, \omega; \mathbf{x}_s), \tag{71}$$

where

$$\Delta V(\mathbf{x}, \omega) = \begin{cases} -\omega^2 K_0 - div\,[\sigma_0 \mathbf{grad}] & \mathbf{x} \in \Delta D\,, \\ 0 & \text{elsewhere} \end{cases} \tag{72}$$

as described in Figure 7, and the domain ΔD is the region of support of $\Delta V(\mathbf{x}, \omega)$. Equation (71) can be rewritten as follows:

$$p(\mathbf{x}_r, \omega; \mathbf{x}_s) = p_P(\mathbf{x}_r, \omega; \mathbf{x}_s) - \omega^2 K_0 \int_{\Delta D} d\mathbf{x}\, G_P(\mathbf{x}_r, \omega; \mathbf{x}) \times p(\mathbf{x}, \omega; \mathbf{x}_s) - \sigma_0 \int_{\Delta D} d\mathbf{x}\, G_P(\mathbf{x}_r, \omega; \mathbf{x}) \times div\,[\mathbf{grad}\; p(\mathbf{x}, \omega; \mathbf{x}_s)]\,. \tag{73}$$

Using the boundary condition that the pressure field $\mathbf{p}$ of the actual data vanishes at the free surface and above the free surface, equation (73) reduces to

$$p(\mathbf{x}_r, \omega; \mathbf{x}_s) = p_P(\mathbf{x}_r, \omega; \mathbf{x}_s) - \sigma_0 \int_{\Delta D} d\mathbf{x}\, G_P(\mathbf{x}_r, \omega; \mathbf{x}) \times div\,[\mathbf{grad}\; p(\mathbf{x}, \omega; \mathbf{x}_s)]\,. \tag{74}$$

We can go a step further by using the Gauss divergence theorem (as we do in Appendix A for the derivation of the Lippmann-Schwinger equation). We can convert the volume integral in equation (74) to a surface integral over the free-surface S' bounding ΔD, as follows:

$$p(\mathbf{x}_r, \omega; \mathbf{x}_s) = p_P(\mathbf{x}_r, \omega; \mathbf{x}_s) - \sigma_0 \oint_{S'} dS(\chi)\, G_P(\mathbf{x}_r, \omega; \mathbf{x}) \times \frac{\partial p(\mathbf{x}, \omega; \mathbf{x}_s)}{\partial n'}, \tag{75}$$

where $\boldsymbol{n}'$ is an outward-pointing normal vector, as depicted in Figure 7. As we did earlier for the representation theorem in section 2, we consider the surface $S' = S'_0 + S'_R$ as the sum of two surfaces: S'_0 is the free surface, and S'_R represents a homogeneous hemisphere of radius R. Since $\partial p(\mathbf{x}, \omega; \mathbf{x}_s)/\partial n'$ is zero on the surface $S_{R'}$, this surface gives a zero contribution to the surface integral in equation (75). Equation (75) becomes

$$p(\mathbf{x}_r, \omega; \mathbf{x}_s) = p_P(\mathbf{x}_r, \omega; \mathbf{x}_s) + \sigma_0 \int_{S'_0} dS(\chi)\, G_P(\mathbf{x}_r, \omega; \mathbf{x}) \times \frac{\partial p(\mathbf{x}, \omega; \mathbf{x}_s)}{\partial n}. \tag{76}$$

Notice that we have used the fact $\mathbf{n}' = -\mathbf{n}$ (see Figures 1 and 7). We can see that equation (76) is equivalent to the representation theorem in equation (6).

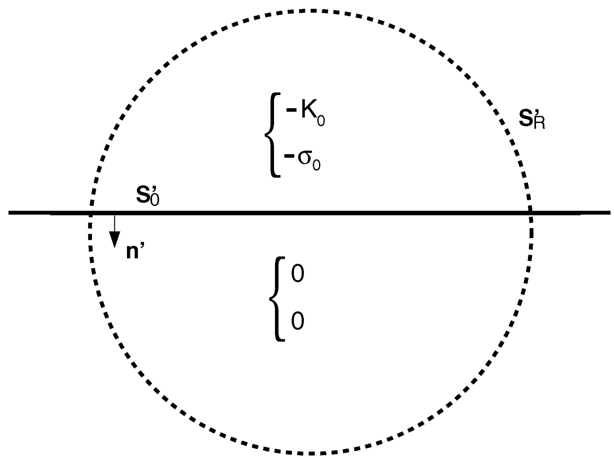

FIG. 7. Geometries of the model associated with the scatterer potentials, $\Delta\mathbf{V}$, defined in equation (72).

If the Lippmann-Schwinger integral equation and the representation theorem are equivalent, then the Kirchhoff and Born series must also be equivalent. Actually, they are. The differences pointed out in our earlier discussion are related to our traditional way of choosing a homogeneous medium as the reference medium. In fact, as in most iterative or series expansion approaches for solving integral equations like those discussed here, the choice of a starting model (here the reference medium) can affect the convergence rate as well as the form of the final solution. Because in practice we work with truncated series or use a small number of iterations due to limited resources, the choice of the reference medium can profoundly affect the practical form of our solutions. So the differences between the Kirchhoff and Born series pointed out here are essentially due to the choice of the homogeneous half-space as the reference medium in the Born case, whereas the choice of the hypothetical medium associated with primaries (see Figure 5) as the reference medium leads to the Kirchhoff series.

Removal of the direct wave

One difference between series (33) and (56) that we did not elaborate on in the previous section is that the Born series requires direct-wave removal, whereas the Kirchhoff does not. For shallow waters (less than 100 m), in which the removal of the direct wave is an issue, equations (16) and (17) are very useful. Let us recall that the direct wave is generally muted by the data by exploiting the separability between direct waves and the rest of the streamer seismic data. In shallow waters, this separability is not attainable; therefore, equations (16) and (17), and the Kirchhoff inverse scattering series solution, which does not require the removal of direct-wave arrivals, will be more suitable for shallow waters than the Born series.

Source signature

The scattering Kirchhoff for attenuating free-surface multiples from streamer data can alternatively be derived from equation (12), which requires the deconvolution of the pressure and vertical particle velocity by the actual source signature used in the seismic acquisition, instead of equation (11). The Kirchhoff series resulting from equation (12) allows the demultiple and designature processes to be combined. This combination is possible because the "triangle relationship" (Amundsen, 2001), which is based on dual-field measurements, can be used to estimate the desired source signature $s(\omega)$ for the deconvolution of pressure and vertical particle velocity.

This triangle relationship defines a theoretical relation among the pressure, the vertical component of particle velocity, and the source signature when the receiver depth level is below the source depth level, $z_r > z_s$. This relationship has been used by Weglein and Secrest (1990) and Osen et al. (1995) to estimate the source wavelet from recordings of pressure and particle velocity, and by Amundsen et al. (1995) to estimate the particle velocity from the pressure when the source wavelets are known. The triangle relationship in the frequency-wavenumber domain (see, e.g., Amundsen, 2001) is

$$\exp[ik_z(z_r - z_s)]G_-(z_s)s(\omega) = ik_z G_+(z_r)P(\boldsymbol{\kappa}, z_r, \omega, \mathbf{x}_s) + i\omega\rho G_-(z_r)V_z(\boldsymbol{\kappa}, z_r, \omega, \mathbf{x}_s) \qquad (z_r > z_s), \tag{77}$$

where

$$G_\pm(z) = 1 \pm \exp(2ik_z z) \tag{78}$$

and where $P(\boldsymbol{\kappa}, z_r, \omega, \mathbf{x}_s)$ and $V_z(\boldsymbol{\kappa}, z_r, \omega, \mathbf{x}_s)$ are the Fourier transform of $p(\chi, z_r, \omega, \mathbf{x}_r)$ and $v_z(\chi, z_r, \omega, \mathbf{x}_r)$, respectively. Here, $G_+(z_r)$ and $G_-(z_r)$ are receiver ghost operators that would be experienced by geophones and hydrophones, respectively, and $G_-(z_{s_i})$ is a source ghost operator.

CONCLUSIONS

We have used the representation theorem to rederive the inverse scattering series for removing free-surface multiples from marine seismic data. We have called this series the "inverse Kirchhoff scattering series" to differentiate it from the currently used inverse Born scattering series, which is derived from the Lippman-Schwinger equation.

The derivation of the inverse Kirchhoff scattering series shows that the inverse scattering series does not necessarily require the removal of the direct wave or a deghosting of the data prior to the demultiple process. This derivation also shows that when dual-field measurements of the pressure field (i.e., the normal gradient of the pressure field in addition to the pressure field itself) are available, computation of the obliquity is not required. Furthermore, the inverse scattering series can be formulated as a designature and demultiple tool along the lines suggested by Amundsen (2001) and Amundsen et al. (2001).

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APPENDIX A

DERIVATION OF THE LIPPMAN-SCHWINGER INTEGRAL EQUATION

Although the Lippman-Schwinger integral equation in equation (45) will hold for most seismic acquisition geometry, it is important to clarify the underlying assumptions as we did for the representation theorem. One way of making this clarification is to rederive the Lippman-Schwinger integral equation from the wave equation. Thus the underlying assumptions will become obvious.

We start with wave equation (42), which governs the wave propagation in the actual medium, i.e.,

$$L(\mathbf{x}, \omega) p(\mathbf{x}, \omega; \mathbf{x}_s) = s(\omega)\delta(\mathbf{x} - \mathbf{x}_s), \qquad \text{(A-1)}$$

and wave equation (37), which governs the wave propagation in the reference medium, i.e.,

$$L_0(\mathbf{x}, \omega) p_0(\mathbf{x}, \omega; \mathbf{x}_s) = s(\omega)\delta(\mathbf{x} - \mathbf{x}_s). \qquad \text{(A-2)}$$

If we multiply equation (A-1) by the pressure $p_0(\mathbf{x}, \omega; \mathbf{x}')$ and integrate over volume D (see Figure 1 for the definition of this volume), we arrive at

$$\int_D d\mathbf{x}' p_0(\mathbf{x}, \omega; \mathbf{x}') L(\mathbf{x}', \omega) p(\mathbf{x}', \omega; \mathbf{x}_s) = s(\omega) p_0(\mathbf{x}, \omega; \mathbf{x}_s). \qquad \text{(A-3)}$$

Similarly, if we multiply this equation by the pressure $p(\mathbf{x}, \omega; \mathbf{x}_s)$, which corresponds to the wave propagation in the actual medium and the integral over volume D, we arrive at

$$\int_D d\mathbf{x}' \; p(\mathbf{x}, \omega; \mathbf{x}') L_0(\mathbf{x}', \omega) p_0(\mathbf{x}', \omega; \mathbf{x}_s) = -s(\omega) p(\mathbf{x}, \omega; \mathbf{x}_s). \qquad \text{(A-4)}$$

The difference between equations (A-3) and (A-4) is

$$\begin{aligned} p(\mathbf{x}, \omega; \mathbf{x}_s) = {} & p_0(\mathbf{x}, \omega; \mathbf{x}_s) + \frac{1}{s(\omega)} \int_D d\mathbf{x}' \; p_0(\mathbf{x}, \omega; \mathbf{x}') \\ & \times V(\mathbf{x}', \omega) p(\mathbf{x}', \omega; \mathbf{x}_s) - \frac{1}{s(\omega)} \oint_S dS(\chi') \\ & \times \sigma_0 \left[p(\mathbf{x}, \omega; \mathbf{x}') \frac{\partial p_0(\mathbf{x}', \omega; \mathbf{x}_s)}{\partial n} \right. \\ & \left. - \; p_0(\mathbf{x}, \omega; \mathbf{x}') \frac{\partial p(\mathbf{x}', \omega; \mathbf{x}_s)}{\partial n} \right], \end{aligned} \qquad \text{(A-5)}$$

where S is the surface bounding domain D. The surface integral in this equation is due to the fact that, mathematically,

$$\begin{aligned} \int_D d\mathbf{x}' p(\mathbf{x}', \omega; \mathbf{x}_s) L_0(\mathbf{x}', \omega) p_0(\mathbf{x}, \omega; \mathbf{x}_s) = {} & \int_D d\mathbf{x}' \\ & \times p_0(\mathbf{x}', \omega; \mathbf{x}_s) L_0(\mathbf{x}', \omega) p(\mathbf{x}, \omega; \mathbf{x}_s) \\ & + \sigma_0 \int_D d\mathbf{x}' div \left[p(\mathbf{x}', \omega; \mathbf{x}) \mathbf{grad} \, p_0(\mathbf{x}', \omega; \mathbf{x}_s) \right. \\ & \left. - \; p_0(\mathbf{x}', \omega; \mathbf{x}) \mathbf{grad} \, p(\mathbf{x}', \omega; \mathbf{x}_s) \right]. \end{aligned} \qquad \text{(A-6)}$$

Using the Gauss divergence theorem, we have obtained equation (A-5).

As we did earlier for the representation theorem in section 2, we consider the surface $S = S_0 + S_R$ as the sum of two surfaces: S_0 is the free surface and S_R represents a hemisphere of radius R. If we let radius R go to infinity, surface $S_{R\to\infty}$ gives a zero contribution to the surface integral in equation (A-5), according to Sommerfeld (1954). Furthermore, both p and p_0 are null at the free surface. Therefore, the surface integral in equation (A-5) is null, and we can obtain the Lippman-Schwinger integral equation in equation (45).